Linguistics and Philosophy
https://doi.org/10.1007/s10988-020-09297-5

ORIGINAL RESEARCH

Counting, measuring, and the fractional cardinalities puzzle

Eric Snyder[1]

Abstract

According to what I call the Traditional View, there is a fundamental semantic distinction between counting and measuring, which is reflected in two fundamentally different sorts of scales: discrete cardinality scales and dense measurement scales. Opposed to the Traditional View is a thesis known as the Universal Density of Measurement: there is no fundamental semantic distinction between counting and measuring, and all natural language scales are dense. This paper considers a new argument for the latter, based on a puzzle I call the Fractional Cardinalities Puzzle: if answers to 'how many'-questions always designate cardinalities, and if cardinalities are necessarily discrete, then how can e.g. '2.38' be a correct answer to the question 'How many ounces of water are in the beaker?'? If cardinality scales are dense, then the answer is obvious: '2.38' designates a fractional cardinality, contra the Traditional View. However, I provide novel evidence showing that 'many' is not uniformly associated with the dimension of cardinality across contexts, and so 'how many'-questions can ask about other kinds of measures, including e.g. volume. By combining independently motivated analyses of cardinal adjectives, measure phrases, complex fractions, and degrees, I develop a semantics intended to defend the Traditional View against purported counterexamples like this and others which have received a fair amount of recent philosophical attention.

Keywords Counting · Measuring · Fractions · Measure phrases · Degrees

This publication was funded by LMU Munich's Institutional Strategy LMUexcellent within the framework of the German Excellence Initiative.

✉ Eric Snyder
E.Snyder@lmu.de

1 Ludwig-Maximilians-Universität München, Munich, Germany

Published online: 26 March 2020

1 The fractional cardinalities puzzle

According to what I call THE TRADITIONAL VIEW, which arguably traces back to at least Frege (1903),[1] there is a fundamental semantic distinction between counting and measuring, which is reflected in the structures of two importantly different sorts of natural language scales. Rothstein (2016, pp. 5–6) summarizes this distinction nicely:

> Counting answers in English the question *how many?* In *three apples*, *three* gives the quantity or size of the sum of apples in terms of the number of individual apples out of which the sum is made up. When we count "one apple, two apples, three apples...", we identify distinct individual apples, and put them in one-to-one correspondence with the natural numbers... In contrast, measuring answers the question *how much?* It involves assigning a quantity an overall value on a dimensional scale, for example the scale of weight, volume, height, cost, and so on. *Three kilos* expresses the property of weighing three kilos on the scale on weight, and *three kilos of apples* denotes sums of apples which weigh three kilos.

Since counting involves determining how many individuated entities belong to a collection, i.e. its cardinality, and since such entities naturally count as units of a certain countable property, ordering cardinalities into a scale results in a *discrete* structure, isomorphic to the natural numbers. In contrast, since measuring involves assigning a numerical value to a substance in virtue of a ratio obtaining between it and a unit of measurement (e.g. kilo) according to some dimension of measurement (e.g. weight), ordering such values into measurement scales results in a *dense* structure, isomorphic to the rationals or reals.

To illustrate, consider the analysis of cardinality expressions, i.e. expressions whose semantic purpose is to count, due to Landman (2004), Scontras (2014) and Rothstein (2016, 2017). On these analyses, count nouns such as 'orange' denote individuated oranges, or ATOMS in the sense of Link (1983), plural nouns such as 'oranges' denote atoms along with sums formed from those atoms, and the semantic function of cardinal modifiers such as 'two' is to count atomic parts. This is reflected in (1), where '$^*\texttt{orange}$' denotes the closure of the atomic oranges under sum-formation, and '#' is a cardinality-function mapping entities to numbers representing their atomic parts.[2]

(1) a. $[\![\text{two}]\!] = \lambda P.\lambda x.\ \#(x) = 2 \wedge P(x)$

b. $[\![\text{two oranges}]\!] = \lambda x.\ \#(x) = 2 \wedge {}^*\texttt{orange}(x)$

Hence, 'two oranges' is a predicate true of sums of oranges having exactly two atomic parts. The resulting picture is as follows, where a, b, and c are atomic oranges, '$\sqcup$' is the sum-operation, and arrows represent the relation of individual parthood.

[1] Frege (1903) argues that the naturals and reals are ontologically distinct, or "form completely separate domains", based on the contention that they answer different kinds of questions. Whereas naturals are discrete and answer 'how many'-questions, reals are dense and answer questions about magnitudes. See Snyder and Shapiro (2016).

[2] The authors mentioned arrive at the denotation in (1a) in different ways. However, these differences are unimportant for present purposes.

$$\begin{tikzcd}
 & a \sqcup b \sqcup c \arrow[rr, dotted, no head] & & \llbracket \text{three oranges} \rrbracket \\
a \sqcup b \arrow[ur] & a \sqcup c \arrow[u] & b \sqcup c \arrow[ul] \arrow[r, dotted, no head] & \llbracket \text{two oranges} \rrbracket \\
a \arrow[u] \arrow[ur] & b \arrow[ul] \arrow[ur] & c \arrow[ul] \arrow[u] \arrow[r, dotted, no head] & \llbracket \text{one orange} \rrbracket
\end{tikzcd}$$

As Krifka (1989) notes, because atomic oranges have no oranges as proper parts, they naturally count as a unit of being an orange, and so evaluate to 1 with respect to #. Consequently, numbers returned by # will be isomorphic to the naturals, and ordering them into a scale results in a necessarily discrete structure. There cannot be, for instance, a sum of oranges having more than two but less than three atomic parts.

In contrast, these same theories analyze measure phrases such as 'two ounces of water' as in (2), where '$\mu_{oz,vol}$' is a MEASURE FUNCTION, or a function from entities to numbers representing a ratio of those entities to a unit named by the measure noun, in this case an ounce, according to some dimension of measurement, in this case volume.

(2) a. $\llbracket \text{ounce} \rrbracket = \lambda P.\lambda n.\lambda x.\ \mu_{oz,vol}(x) = n \wedge P(x)$

b. $\llbracket \text{ounce of water} \rrbracket = \lambda n.\lambda x.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(x)$

c. $\llbracket \text{two ounces of water} \rrbracket = \lambda x.\ \mu_{oz,vol}(x) = 2 \wedge \texttt{water}(x)$

Hence, 'two ounces of water' is a predicate true of quantities of water measuring two ounces. Since the numbers in (2) represent a ratio of a quantity of water to an ounce of water, and since such ratios needn't correspond to some integer multiple of one ounce, ordering them into a scale results in a necessarily dense structure, isomorphic to the rationals or reals.[3]

Opposed to the Traditional View is a thesis Fox and Hackl (2007) call THE UNIVERSAL DENSITY OF MEASUREMENT (UDM): there is no fundamental semantic distinction between counting and measuring, and all natural language scales are dense. Fox and Hackl's primary argument for UDM is that it readily explains contrasts like (3).

[3] This is opposed to a view, defended by e.g. Schwarzschild (2002, 2005) and Bale (2008), on which the function of a numeral within a measure phrase is to *count* discrete, atomic *degrees*, denoted by the measure noun, as suggested in (i).

(i) a. $\llbracket \text{ounce} \rrbracket = \lambda d \in domain(vol).\ d = oz$

b. $\llbracket \text{two ounces} \rrbracket = \lambda d.\ \#(d) = 2 \wedge {}^{*}\texttt{ounce}(d)$

c. $\llbracket \text{two ounces of rice} \rrbracket = \lambda x.\ \texttt{two-ounces}(vol(x)) \wedge \texttt{rice}(x)$

However, such a view appears to have difficulty explaining contrasts like those in (ii), which have been used to argue that whereas so-called "atomizer nouns" such as 'grain' form genuinely atomic predicates, measure nouns such as 'ounce' do not (Scontras 2014).

Context: Pointing at two grains of rice and a bowl containing two ounces of rice on a table.

(ii) a. Those {grains/??ounces} of rice are for the experiment.

b. Both of the {grains/??ounces} of rice are for the experiment.

Another potential issue for this view concerns non-whole numerals occurring in measure phrases, such as e.g. '2.38 ounces of water'. The problem is that it is hard to see how '2.38' could represent the result of *counting* atoms of any sort, as it is definitional of x's being an atomic P that x cannot be divided into proper P-parts (Krifka 1989).

(3) a. John has very few children. He only has *three*.
b. ?? John has very few children. He only has more than *three*.

Suppose cardinal 'three' has a lower-bounded meaning, so that 'John has three children' is true just in case he has *at least* three children. Assume further that 'only' rules out all salient alternatives except for the one explicitly mentioned. Then if cardinality scales are discrete, (3a) should rule out John's having more than three children, thus implying that he has exactly three children. By similar reasoning, (3b) should acceptably imply that John has exactly four children, contrary to fact. On the other hand, if cardinality scales are dense, so that there are salient alternative cardinalities between three and four, then (3b) would imply that John has exactly three children. Since this is inconsistent with what is explicitly stated, (3b) is anomalous.

The focus of this paper is not with Fox and Hackl's arguments. Rather, it is with a different kind of argument for UDM, one which is arguably more direct and intuitively compelling. It comes in response to a certain semantic puzzle involving cardinality expressions, which takes the form of an inconsistent triad. The first premise is Discreteness.

DISCRETENESS: Ordering cardinalities into a scale results in a discrete structure, isomorphic to the natural numbers.

Discreteness is an assumption implicit in nearly all extant analyses of cardinality expressions, including, but not limited to, those mentioned above.[4] And though there are numerous characterizations of cardinalities available—e.g. as real numbers, degrees, tropes, or equivalence classes of equinumerous concepts[5]—insofar as they represent how many individuated entities belong to a collection, Discreteness is seemingly inevitable.

The second premise of the puzzle is Inquiry.

INQUIRY: Answers to 'how-many'-questions always designate cardinalities.

To my knowledge, Inquiry is implicit in all extant analyses of 'how many'-questions. To illustrate, consider Rullman (1993)'s analysis in (4), where '$?n$' roughly translates as "what's the maximal number n ...".[6]

(4) $[\![\text{how many}]\!] = \lambda P.\lambda Q.?n.\exists x.\,\#(x) = n \wedge P(x) \wedge Q(x)$

As above, '#' is a cardinality-function mapping pluralities to numbers representing their atomic parts. Accordingly, the 'how many'-questions in (5a) and (6a) might be analyzed respectively as (5b) and (6b).

(5) a. How many oranges are on the table?
b. $?n.\exists x.\,\#(x) = n \wedge {}^{*}\texttt{orange}(x) \wedge \texttt{on-table}(x)$

[4] These include Frege (1884)'s well-known analysis, quantificational analyses like that of Barwise and Cooper (1981), variations of that analysis like Breheny (2008), and degree-based analyses like that of Hackl (2000) and Kennedy (2015).

[5] See Balcerak-Jackson and Penka (2017).

[6] This characterization is intended to be neutral between various analyses of questions, including e.g. Karttunen (1977) and Groenendijk and Stokhof (1991).

(6) a. How many ounces of water are in the beaker?

b. $?n.\exists x.\ \#(x) = n \wedge {}^{*}\texttt{ounce-of-water}(x) \wedge \texttt{in-beaker}(x)$

Thus, whereas (5a) would inquire into the cardinality of atomic oranges on the table, (6a) would inquire into the cardinality of the atomic ounces of water in the beaker.[7]

This leads to the final premise, Counterexamples.

> COUNTEREXAMPLES: Some correct answers to 'how many'-questions designate entities which do not correspond structurally to natural numbers.

The motivation for Counterexamples comes directly from examples such as (7a,b), inspired by Solt (2009).

(7) a. How many oranges are on the table? $2\frac{1}{2}$.

b. How many ounces of water are in the beaker? 2.38.

As we will see, (7a) in particular has received a fair amount of recent philosophical attention. However, (7b) illustrates a similar point. By Inquiry, '$2\frac{1}{2}$' in (7a) and '2.38' in (7b) designate cardinalities, i.e. the sorts of things ranged over by 'n' in (5b) and (6b). By Discreteness, those cardinalities are discrete, and so correspond structurally to natural numbers. Yet $2\frac{1}{2}$ and 2.38 are obviously *not* natural numbers.

This leads to what I call THE FRACTIONAL CARDINALITIES PUZZLE (FCP): How can (7a,b) be correct answers to the 'how many'-questions posed if they designate fractional cardinalities, and yet fractional cardinalities are incoherent by the lights of the Traditional View?

The argument for UDM I want to consider is just this: adopting UDM offers a straightforward and independently motivated solution to FCP. If cardinality scales are dense, then it is hardly surprising that we can answer 'how many'-questions with expressions designating non-discrete cardinalities. If so, then examples like (7a,b) provide direct and intuitively compelling evidence that all natural language scales are dense.

Of course, this constitutes an argument for UDM only if an analysis consistent with the possibility of fractional cardinalities can be produced. Following a suggestion from Kennedy and Stanley (2009), one might thus adopt Krifka (1989)'s semantics for count nouns. Basing it on classifier languages, Krifka analyzes all nouns as basically mass, denoting quantities of stuff. Count nouns are differentiated in virtue of having a certain classifier-like element built into their meanings, which Krifka calls a NATURAL UNIT (NU). Intuitively, NU determines a ratio of stuff of the kind named by the noun to a contextually-determined natural unit of that same kind.[8] For example, the count noun 'orange' contains NU as part of its meaning, which determines a ratio between a particular amount of orange-stuff and a contextually-determined natural unit of orange. This is given in (8a), where 'NU' is a function mapping quantities of orange to real numbers.

[7] See fn.3.

[8] See Krifka (1989, pp. 82–83) for a discussion of constraints on NU, as well as Rothstein (2017, Ch. 3) for a more general discussion of Krifka's semantics and its relation to other theories of nouns.

(8) a. $[\![\text{orange}]\!] = \lambda n.\lambda x.\ \texttt{orange}(x) \wedge NU(\texttt{orange})(x) = n$

b. $[\![2\frac{1}{2}\ \text{oranges}]\!] = \lambda x.\ \texttt{orange}(x) \wedge NU(\texttt{orange})(x) = 2.5$

Since NU determines a ratio, nothing prevents it from returning a *non-whole* real number in a situation where, for instance, there are two whole oranges and one half orange on the table. Indeed, assuming that the relevant natural unit in such a scenario is a typical whole orange, NU will return the real number 2.5, as suggested in (8b). Given Inquiry, this also designates a *cardinality* in response to a 'how many'-question like (7a).

Thus, Krifka's analysis affords a way of coherently denying Discreteness if Inquiry is adopted and cardinalities are identified with real numbers. Of course, an analysis of 'how many'-questions consistent with the possibility of fractional cardinalities is also required. As we'll see in Sect. 2, one could adopt a modified version of Rett (2008)'s analysis to this end. On Rett's analysis, 'how many'-questions ask about degrees of cardinality, where degrees more generally are identified with real numbers. By generating cardinalities in the manner suggested by Krifka's analysis, it thus becomes possible to ask about their positions on a dense cardinality scale. For similar reasons, it is also possible to inquire into non-discrete cardinalities using measure phrases.

On the other hand, denying Discreteness in this manner provides evidence for UDM only if the resulting analysis is empirically *adequate*. Following Snyder and Barlew (2019), I will argue that Krifka's analysis is problematic because it has no obvious semantic means of capturing a certain ambiguity in '$2\frac{1}{2}$ oranges'. More specifically, Snyder and Barlew argue that '$2\frac{1}{2}$ oranges' is ambiguous in a way resembling so-called CONTAINER PHRASES like 'two glasses of water', which are known to be ambiguous between an INDIVIDUATING INTERPRETATION (II) paraphrased in (9b), and a MEASURE INTERPRETATION (MI) paraphrased in (9c).[9]

I-Context: Mary has a strange way of heating water for coffee: she fills two glasses with water, and places those glasses into boiling soup.

M-Context: Mary is making soup. The recipe calls for two glassfuls of water. Estimating, Mary pours water directly from the tap into the soup.

(9) a. Mary put two glasses of water in the soup.

b. Mary put two glasses filled with water in the soup.

c. Mary put two glassfuls of water in the soup.

In the I(ndividuating)-Context, we are *counting* two atomic glasses, which happen to be filled with water. In the M(easure)-Context, we are *measuring* an amount of water in terms of a non-canonical, or AD HOC, glass-unit, i.e. how much water would fill some contextually salient glass.

Snyder and Barlew argue that (10a) is similarly ambiguous, as shown by applying the same diagnostics responsible for revealing I/M-ambiguities in container phrases.

I-Context: Mary has made punch for the party, and wants to decorate it. She thinks floating fruit would look nice. She drops several apples, two oranges, and one half orange into the punch.

[9] See Landman (2004), Rothstein (2009, 2017), Partee and Borschev (2012), Scontras (2014), and Snyder and Barlew (2016, 2019).

M-Context: Mary is making punch. The recipe calls for $2\frac{1}{2}$ oranges worth of orange pulp. Estimating, she pours some prepackaged orange pulp directly from a carton into the punch.

(10) a. Mary put two and a half oranges in the punch.
b. Mary put two oranges and one half orange in the punch.
c. Mary put two and a half oranges worth of orange in the punch.

Thus, (10a) is ambiguous between an II paraphrased in (10b), where we are counting two oranges and one half orange, and a MI paraphrased in (10c), where we are measuring quantities of orange in terms of an ad hoc orange-unit, i.e. how much orange constitutes some contextually salient orange.

The trouble is that since 'orange' denotes a ratio of quantities of orange to a natural unit on Krifka's analysis, and since the same natural unit is presumably relevant in *both* contexts provided, namely a typical whole orange,[10] Krifka's semantics predicts that (10a) should have the same truth-conditions in both contexts: it is true just in case the ratio of orange-stuff on the table to a typical whole orange is the real number 2.5. While this is certainly true in both contexts, the analysis offers no obvious explanation for numerous semantic *differences* between IIs and MIs. For example, (11) is acceptable in I-Context provided, but not in the M-Context.

(11) Mary put two and a half {beautiful/large} oranges in the punch.

Additional, similar contrasts show that '$2\frac{1}{2}$ oranges' is genuinely ambiguous in a way that is not obviously recoverable within Krifka's semantics.

This suggests that a different solution to FCP is called for. I will argue that the actual culprit is not Discreteness, but *Inquiry*. On the analyses of Rett (2008) and Solt (2009, 2014), 'many' and 'much' are gradable adjectives which differ in one important respect: whereas 'much' can be associated with different dimensions of measurement in different contexts, 'many' always encodes cardinality. As a result, 'how many'-questions always inquire into cardinalities. However, consider the 'how many'-question in (12a), which is similarly ambiguous between an II in (12b) and a MI in (12c).

I-Context: Similar to (9). Knowing Mary's procedure, John asks:

M-Context: Similar to (9). Knowing the recipe, John asks:

(12) a. How many glasses of water did you put in the soup?
b. How many glasses filled with water did you put in the soup?
c. How many glassfuls of water did you put in the soup?

We saw above that whereas IIs involve *counting* individuated glasses, MIs instead involve *measuring* an amount of water in terms of an ad hoc glass-unit. Since cardinalities represent the result of counting on the Traditional View, John's question is plausibly interpreted as inquiring into a cardinality only in the *individuating* context.

[10] As is needed to guarantee that the correct ratio obtains in both contexts.

In the measure context, it is instead naturally interpreted as inquiring into the *volume* of water Mary put into the soup. If so, then answers to 'how many'-questions do not *always* specify cardinalities, contra Inquiry.

Thus, on the view defended here, 'many' is like 'much' in that *both* can be associated with different dimensions of measurement in different contexts. In individuating contexts, 'many' is naturally associated with the dimension of cardinality, and so answers to 'how many'-questions count the atomic parts of pluralities. In measure contexts, however, 'many' will be associated with a different dimension of measurement such as volume, and so answers to 'how many'-questions in such contexts will specify such a measure.

An immediate consequence of the view is that there is no guarantee that the 'how many'-questions in (7a,b) are asking about *cardinalities*. In fact, since (7a) is I/M-ambiguous, it follows that '$2\frac{1}{2}$ oranges' specifies a cardinality only in *individuating* contexts. Yet, given Snyder and Barlew's analysis, the II of '$2\frac{1}{2}$ oranges' does not require postulating fractional cardinalities, as again we are counting *two* whole oranges and *one* half orange. In (7b), on the other hand, the 'how many'-question is naturally interpreted as inquiring into the *volume* of water in the beaker, and '2.38' is naturally interpreted as specifying that volume. Hence, fractional cardinalities are not required in either case, and so (7a,b) present no special difficulties for the Traditional View, nor do they provide independent support for UDM.

The rest of the paper is structured as follows. In Sect. 2, I sketch the hybrid Krifka-Rett semantics mentioned above, as well as its empirical shortcomings. In Sect. 3, I consider and debunk three arguments for Inquiry, appealing to novel evidence to do so. This opens the way for an alternative semantics for examples like (7a,b). This is provided in Sect. 4, where I show how combining a modified version of Rett's semantics for 'many', along with independently motivated analyses of degrees, measure phrases, and fractions predicts intuitively correct truth-conditions for those and other examples consistent with the Traditional View.

The resulting analysis depends importantly on the contention that 'many', like 'much', can select for different measures in different contexts. Thus, the question arises as to how exactly 'much' and 'many' select different measures, and what constraints, if any, are operative. I conclude the paper by addressing these questions in Sect. 5, where, following recent work by Wellwood (2018, 2019), it is suggested that 'many' and 'much' are *allomorphs*, where 'many' decomposes into 'much' plus plural morphology. Measure selection is then constrained by two general principles so as to reflect differences in *domain structures*. Specifically, atomic domains are measured by cardinality, while non-atomic domains select other measures, such as volume.

2 The counting oranges puzzle

(7a) is due originally to Salmon (1997), who uses it to formulate a puzzle which has received a fair amount of recent philosophical attention.[11]

[11] See Kennedy and Stanley (2009), Liebesman (2015, 2016), Nicolas (2016), Balcerak-Jackson and Penka (2017), Carrara and Lando (2017) and Snyder and Barlew (2019).

(7a) How many oranges are on the table? $2\frac{1}{2}$.

Suppose there are three oranges on the table. I take one of them, cut it in half, eat one of the halves, and set the remaining half back on the table. Intuitively, the correct answer to the 'how many'-question in (7a) is the answer provided: $2\frac{1}{2}$. But a half orange is either an orange or it isn't. If it is, then there are three oranges on the table. And if it isn't, there are only two. In either case, we do not get the intuitively correct answer. Snyder and Barlew (2019) call this THE COUNTING ORANGES PUZZLE (COP).

Salmon's solution to COP is to make counting property-relative, so that counting oranges corresponds to *measuring* orange-stuff relative to some contextually-determined property, in this case being a typical whole orange. Thus, in the context described, the different quantities of orange receive different measurements, corresponding to different ratios: the ratio of whole oranges to a typical whole orange is 2:1, and the ratio of the half orange to a typical whole orange is .5:1. Adding these together, we get the intuitively correct answer to (7a): 2.5. Similar solutions have been suggested or proposed by Kennedy and Stanley (2009) and Liebesman (2015, 2016).

As Kennedy and Stanley observe, this solution is strikingly similar to one suggested by Krifka (1989)'s semantics for count counts. As mentioned, Krifka analyses 'orange' as denoting a ratio of orange-stuff to a natural unit of orange, given in terms of a real number.

(8b) $[\![2\frac{1}{2} \text{ oranges}]\!] = \lambda x.\ \texttt{orange}(x) \wedge NU(\texttt{orange})(x) = 2.5$

The suggestion is that we are not *counting* atomic parts in the context of COP, but rather *measuring* how much orange-stuff is on the table, relative to a typical whole orange. Since that ratio is 2.5:1, it's hardly surprising that the correct answer to the 'how many'-question posed designates a fractional cardinality, assuming Inquiry.

Of course, as a potential solution to COP, we also need an analysis of 'how many'-questions consistent with the possibility of fractional cardinalities. To this end, one might adopt a variant of Rett (2008)'s analysis. According to the latter, 'many' and 'much' are gradable adjectives having the denotation in (13), where D is a set of degrees, identified with real numbers, and 'ℓ' is a function measuring the length of an interval of degrees.[12]

(13) $[\![\text{many/much}]\!] = \lambda D_{<d,t>}.\lambda d.\ \ell[D] = d$

Thus, 'many' and 'much' are degree-modifiers: they map a set of degrees to a set of degrees. Where they differ is the set of degrees measured by ℓ, which is generated via a type-shifting principle Rett calls "QUANTITY". This is defined in (14a), where 'μ' is a contextually-determined measure function.

(14) a. $\text{QUANTITY} = \lambda P.\lambda d.\lambda Q.\exists x.\ P(x) \wedge Q(x) \wedge \mu(x) = d$

b. $\text{QUANTITY}([\![\text{oranges}]\!])([\![\text{on the table}]\!]) = \lambda d.\exists x.\ \texttt{oranges}(x) \wedge \texttt{on-table}(x) \wedge \mu_{\#}(x) = d$

c. $\text{QUANTITY}([\![\text{water}]\!])([\![\text{in the beaker}]\!]) = \lambda d.\exists x.\ \texttt{water}(x) \wedge \texttt{in-beaker}(x) \wedge \mu_{vol}(x) = d$

[12] For a precise characterization, see Rett (2008, pp. 41–42).

With plural nouns such as ‘oranges’, μ is interpreted as a cardinality function, thus generating a set of degrees of cardinality which can combine with ‘many’. Because those degrees represent the atomic parts constituting a plurality, they are necessarily discrete. In contrast, with mass nouns such as ‘water’, μ will be interpreted as some other kind of measure, e.g. volume, thus generating a dense set of degrees of that sort, which can then combine with ‘much’. The function of both ‘many’ and ‘much’, then, is to measure the length of the resulting scalar intervals.

Since cardinalities measured by ℓ are assumed to be discrete, Rett’s analysis presupposes Discreteness. However, suppose we were to drop this assumption. Specifically, suppose we instead adopt Krifka (1989)’s analysis of count nouns, so that e.g. ‘oranges’ denotes a measure of orange-stuff. Also, suppose with Krifka that the numerical argument of count nouns gets existentially bound in the absence of overt numerals, as is presumably the case with e.g. ‘many oranges’. Thus, in the ‘how many’-question in (7a), ‘oranges’ receives the denotation in (15).

(15) $[\![\varnothing \text{ oranges}]\!] = \lambda x.\exists n.\ \texttt{orange}(x) \wedge NU(\texttt{orange})(x) = n$

As a predicate, this is of the right type to combine with Rett’s QUANTITY.

(16) $\text{QUANTITY}([\![\varnothing \text{ oranges}]\!])([\![\text{on the table}]\!] = \lambda d.\exists x.\exists n.\ \texttt{orange}(x) \wedge NU(\texttt{orange})(x) = n \wedge \texttt{on-table}(x) \wedge \mu(x) = d$

Because n here ranges over real numbers, and since degrees are assumed to be real numbers, the result is a set of degrees representing the ratio of orange-stuff on the table to a natural-unit of orange. As before, since that ratio needn’t correspond to a positive integer, the resulting set can be dense. Finally, assuming with Rett that ‘many’ lexically encodes cardinality, we would thus have a dense set of *cardinalities*.

As a set of degrees, (16) can then combine with ‘many’ in (13) to return a set of degrees measured by ℓ.

(17) $[\![\text{many oranges on the table}]\!] = \lambda d'.\ \ell[\lambda d.\exists x.\exists n.\ \texttt{orange}(x) \wedge NU(\texttt{orange})(x) = n \wedge \texttt{on-table}(x) \wedge \mu(x) = d] = d'$

Consequently, ‘many oranges on the table’ has a meaning suitable to combine with other degree morphemes, including e.g. ‘very’ and ‘how’. On Rett’s analysis, the latter is analyzed as (18), where ‘β’ is a relation holding between a degree and a set of propositions, i.e. a question.[13]

(18) $[\![\text{how}]\!] = \lambda \beta_{<d,<<s,t>,t>>}.\lambda p.\exists d[\beta(p)(d)]$

Ultimately, this can combine with (17) and the head of the ‘wh’-clause to generate a meaning for the ‘how many’-question in (7a) given in (19), where ‘$w_{@}$’ designates the actual world.[14]

(19) $\lambda p.\exists d'.\ p(w_{@}) \wedge p = \lambda w.\ \ell[\lambda d.\exists x.\exists n.\ \texttt{orange}(w)(x) \wedge NU(\texttt{orange})(x) = n \wedge \texttt{on-table}(w)(x) \wedge \mu(x) = d] = d'$

[13] See Karttunen (1977).

[14] See Rett (2008, Ch. 2) for details.

Consequently, the question in (7a) denotes a set of true propositions of the following form: there are d-many oranges on the table, where d is a degree of cardinality representing a ratio of orange-stuff to a natural unit of orange. Again, because the degrees measured by ℓ are dense, it's hardly surprising that the correct answer to (7a) designates a fractional cardinality.

A similar analysis can be given for (7b), assuming the analysis of measure phrases from Sect. 1, shared by Krifka.

(7b) How many ounces of water are in the beaker? 2.38.

According to the latter, recall, '2.38' specifies a non-whole ratio of water in the beaker to an ounce of water. Thus, we can similarly obtain a set of degrees measuring the volume of water in the beaker via QUANTITY in the manner suggested in (20).

(20) a. $[\![\text{ounce of water}]\!] = \lambda n.\lambda x.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(x)$

b. $[\![\varnothing \text{ ounce of water}]\!] = \lambda x.\exists n.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(x)$

c. $\text{QUANTITY}([\![\varnothing \text{ ounce of water}]\!])([\![\text{in the beaker}]\!]) =$
$\lambda d.\exists x.\exists n.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(x) \wedge \texttt{in-beaker}(x) \wedge \mu(x) = d$

Continuing to assume that 'many' encodes cardinality, combining (20c) with 'many' will map the degrees in (20c) onto a dense cardinality scale, thereby determining the length of the original interval, as suggested in (21).

(21) $[\![\text{many ounces of water in the beaker}]\!] = \lambda d'.\ \ell[\lambda d.\exists x.\exists n.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(x) \wedge \texttt{in-beaker}(x) \wedge \mu(x) = d] = d'$

This in turn can combine with 'how' and the head of the 'wh'-clause to generate a meaning for the question in (7b) given in (22).

(22) $\lambda p.\exists d'.\ p(w_{@}) \wedge p = \lambda w.\ \ell[\lambda d.\exists x.\exists n.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(w)(x) \wedge \texttt{in-beaker}(w)(x) \wedge \mu(x) = d] = d'$

According to (22), the 'how many'-question in (7b) denotes a set of true propositions of the following form: there are d-many ounces of water in the beaker, where d is a degree of cardinality representing the ratio of water in the beaker to a canonical ounce. As before, since (i) ℓ measures an interval of degrees, (ii) those degrees are *dense*, and (iii) the degree measuring the length of that interval is a *cardinality*, it's little wonder that the correct answer to (7b) specifies a fractional cardinality.

The resulting hybrid semantics not only offers a seemingly plausible solution to COP, it also vindicates UDM by coherently denying Discreteness. As Salmon suggests, we are not counting atomic oranges in the context of COP. Rather, we are measuring an amount of orange-stuff relative to a natural unit of orange. This corresponds to a cardinality, which the 'how many'-question in (7a) asks about. Similarly, we are not counting atomic ounces of water in (7b), but measuring an amount of water relative to a canonical ounce. This again corresponds to a cardinality, assuming Inquiry. Thus, fractional cardinalities are coherent in virtue of representing a ratio of stuff to either a contextually-determined natural unit associated with a count noun or else a canonical unit named by a measure noun. Call this THE KRIFKA-INSPIRED ANALYSIS (KIA).

Despite it's initial appeal, I'm going to argue that KIA is not the best solution to COP or FCP. That's because, following Snyder and Barlew (2019), I will argue that it has no obvious semantic means for explaining why '$2\frac{1}{2}$ oranges' is ambiguous in way similar to 'two glasses of water'. Ultimately, this suggests that an alternative analysis is called for.

2.1 The challenge to KIA

Snyder and Barlew (2019) (henceforth 'S&B') develop certain diagnostics for teasing apart I(ndividuating)/M(easure)-ambiguous phrases more generally, and apply those diagnostics to reveal that '$2\frac{1}{2}$ oranges' is similarly ambiguous. For example, one diagnostic follows Rothstein (2009), who observes that container phrases are acceptable with the suffix '-ful' in measure contexts but not individuating contexts.

I-Context: Similar to (9). John says: "I noticed some glasses in the soup. How many glasses of water did you put in it?"

M-Context: Similar to (9). John says: "I noticed some water in the soup. How many glasses of water did you put in it?"

(23) Mary: "Two glassfuls." (IC: X, MC: ✓)

According to Rothstein, that's because the semantic effect of '-ful' is to transform a container noun such as 'glass' into a measure noun measuring substances according to an ad hoc glass-unit, thus leading to MIs.

As S&B point out, '-ful' is plausibly a special case of 'worth' in this respect.[15] Generally speaking, 'worth' forces MIs, though it is less restrictive than '-ful', as '-ful' is only acceptable with container nouns. Hence, substituting 'glasses worth' for 'glassful' in (23) delivers the same contrast. Now consider a parallel case involving '$2\frac{1}{2}$ oranges'.

I-Context: There are three whole oranges on the table. Mary takes one, cuts it in half, discards one of the halves, and sets the remaining half back on the table. John says: "I noticed some fruit on the table. How many oranges are on the table?"

M-Context: Similar to (10). John, who knows the punch recipe, says: "I noticed you were making punch. How many oranges did you put in it?"

(24) Mary: "Two and a half oranges worth." (IC: X, MC: ✓)

As before, (24) is acceptable in the measure context but not the individuating context. This makes sense if the latter imposes an II, the former a MI, and if 'worth', like '-ful', transforms the meaning of 'glass' or 'orange' into a measure noun. Note that the I-Context provided here is just the COP scenario, thus revealing that it imposes an *II* of '$2\frac{1}{2}$ oranges'.

As a second illustration, Rothstein shows that deictic plural pronouns such as 'those' are acceptable in individuating but not measure contexts.

[15] See also Rothstein (2017).

I-Context: Similar to (9). Pointing at the soup, Mary says:

M-Context: Similar to (9). Pointing at the soup, Mary says:

(25) Those are two glasses of water. (IC: ✓, MC: X)

As Rothstein explains, deictic 'those' refers to a salient plurality of individuated objects, i.e. atoms. Since we count atoms, (25) makes sense if the II of 'two glasses of water' involves counting atomic glasses, which are present in individuating contexts, while the MI instead involves measuring an amount of water, where atoms are absent.

We see the same contrast with '$2\frac{1}{2}$ oranges', as S&B observe.

I-Context: Similar to (24). Pointing at the table, Mary says:

M-Context: Similar to (10). Pointing at the punch, Mary says:

(26) Those are $2\frac{1}{2}$ oranges. (IC: ✓, MC: X)

Again, this makes sense if 'those' references some salient plurality of individuated objects which are present in the I-Context but not in the M-Context. According to S&B, this plurality consists of *three* such objects, namely two whole oranges and a single half orange.

Ultimately, this ambiguity in '$2\frac{1}{2}$ oranges' is problematic for KIA, for two importantly different reasons. First, KIA has no obvious semantic means for explaining the various contrasts noted between IIs and MIs. That's because in both contexts given, the relevant natural unit is presumably a typical whole orange, and so KIA predicts that (10a) should have the same truth-conditions in both contexts: it is true just in case the ratio of orange-stuff to a typical whole orange is 2.5 to 1. Since this is in fact the case in both contexts given, there is no evident semantic reason why, for example, (26) should be acceptable in one context but not the other.

Secondly, KIA apparently predicts that (10a) should have the *wrong* interpretation in the context of COP.

(10a) Mary put two and a half oranges in the punch.

(10b) Mary put two oranges and one half orange in the punch.

(10c) Mary put two and a half oranges worth of orange in the punch.

That's because the meaning KIA attributes to '$2\frac{1}{2}$ oranges' is most naturally paraphrased as the MI in (10c): '$2\frac{1}{2}$ oranges' denotes quantities of orange *measuring* a certain amount, in terms of a natural unit. Yet we just saw that COP requires an *individuating* interpretation, paraphrased as (10b). To be clear, the problem is not that KIA predicts (10a) to be false in that scenario. Rather, the problem is that it predicts (10a) to be true, but *for the wrong reason*: it is not true in virtue of the correct *ratio* obtaining, but instead in virtue of there being individuated things available to *count*.[16]

[16] It has been suggested that this problem can be overcome if, following Link (1983), we sortally distinguish two domains, atoms and mass quantities, allowing 'orange' to denote either in different contexts. In that case, the II would presumably result if 'orange' denotes atoms, the MI if it instead denotes mass quantities. There are two problems with this suggestion, however. First, it's not at all clear how the II of (10a) would distinguish atomic oranges from atomic *half* oranges. Secondly, it appears to be in conflict with Krifka's original motivation for analyzing count nouns as *uniformly* denoting quantities of stuff, similar to nouns in classifier languages.

This suggests that a different analysis of '$2\frac{1}{2}$ oranges' is in order. Ideally, it would not only capture the semantic differences between the II and MI, it would also relate them in a natural way. As we will see in the next section, S&B's analysis purports to do just that. According to it, the II arises if '$2\frac{1}{2}$ oranges' is interpreted as a predicate true of atomic parts, while the MI arises if 'orange' is shifted into a reflexivized measure noun. We will see that while this affords a plausible solution to COP, it offers no obvious solution to FCP: if '$2\frac{1}{2}$ oranges' has a MI according which '$2\frac{1}{2}$' specifies a *measure* of orange-stuff relative to an ad hoc orange-unit, then how can '$2\frac{1}{2}$ oranges' correctly answer a *how many*-question in measure contexts?

2.2 An alternative analysis

S&B's analysis of '$2\frac{1}{2}$ oranges' builds on previous analyses of I/M-ambiguities due to Rothstein (2009) and Scontras (2014). According to the latter, the II of (9a) arises by shifting the lexical meaning of 'glass'—a predicate true of atomic glasses—into a relational noun denoting glasses bearing a certain contextually-supplied relation to a substance contained.

(9a) Mary put two glasses of water in the soup.

Formally, this can be seen as arising from a type-shifting principle S&B call RELATIONAL NOUN SHIFT (RNS), where free 'R' signifies that its interpretation is provided by context.

(27) a. $\text{RNS} = \lambda P.\lambda Q.\lambda x.\exists y.\ P(x) \wedge Q(y) \wedge R(x, y)$

b. $\text{RNS}([\![\text{glass}]\!])([\![\text{of water}]\!]) = \lambda x.\exists y.\ \texttt{glass}(x) \wedge \texttt{water}(y) \wedge R(x, y)$

With container phrases, 'R' is interpreted as 'being filled with',[17] and so (9a) will be true if there is a plurality consisting of two atomic glasses, each of which is filled with water and is in the soup.

In contrast, MIs result from shifting the lexical meaning of 'glass' into a measure noun relating substances to their measures given in terms of an ad hoc glass-unit. This results from a type-shifting operation S&B call THE UNIVERSAL MEASURER (UM). UM is given in (28), where 'μ_P' is a contextually-determined ad hoc measure based on P.

(28) $\text{UM} = \lambda P.\lambda Q.\lambda n.\lambda x.\ Q(x) \wedge \mu_P(x) = n$

For example, combining UM with 'glass' and 'of water' results in a measure of quantities of water based on an ad hoc glass-unit.

(29) a. $\text{UM}([\![\text{glass}]\!])([\![\text{of water}]\!]) = \lambda n.\lambda x.\ \texttt{water}(x) \wedge \mu_{glass,vol}(x) = n$

b. $[\![\text{two glasses of water}]\!] = \lambda x.\ \texttt{water}(x) \wedge \mu_{glass,vol}(x) = 2$

Assuming 'two' functions as a numeral specifying that measure, (9a) will be true if there is a quantity of water in the soup measuring two glassfuls.

Ultimately, S&B derive truth-conditions for (10a) in a similar manner.

[17] Cf. Partee and Borschev (2012).

(10a) Mary put two and a half oranges in the punch.

Let's begin with the MI. According to S&B, this arises from shifting the lexical meaning of 'orange' into that of a *reflexivized* measure noun, i.e. a noun which measures the substance named by the noun in terms of an ad hoc unit *of the same kind*. This is obtained by reflexivizing UM, resulting in THE REFLEXIVIZED UNIVERSAL MEASURER (RUM) given in (30a), where 'γ' is a "grinding"-like operator returning the (possibly proper) material parts of Ps.

(30) a. RUM = $\lambda P.\lambda n.\lambda x.\ \gamma(P)(x) \wedge \mu_P(x) = n$
b. $\gamma = \lambda P.\lambda x.\exists y.\ x \sqsubseteq y \wedge P(y)$

Thus, applying RUM to 'oranges' returns a predicate true of material parts of oranges measuring a certain amount, where the relevant ratio is presumably given in terms of the volume of the parts of some salient orange.

(31) a. RUM($[\![$oranges$]\!]$) $= \lambda n.\lambda x.\ \gamma(\mathtt{oranges})(x) \wedge \mu_{orange,vol}(x) = n$
b. $[\![$two and a half oranges$]\!] = \lambda x.\ \gamma(\mathtt{oranges})(x) \wedge \mu_{orange,vol}(x) = 2.5$

As before, assuming that numerals specify a measure on MIs, (10a) will be true if the ratio of orange-stuff on the table to an ad hoc orange-unit is 2.5, i.e. if there are two and a half oranges *worth of orange* on the table. This would be true if, for instance, there were two and a half oranges worth of orange slices on the table, each from possibly different oranges, or indeed if we were to take the two whole oranges and single half orange from the original COP scenario and pulverize them into scattered orange pulp.

Deriving intuitively correct truth-conditions for the II is somewhat more complicated, however, due to the mixed fraction 'two and a half'. According to S&B, on the II 'and' in (10a) is an instance of CUMULATIVE (or "non-boolean") CONJUNCTION, as opposed to PROPOSITIONAL (or "boolean") CONJUNCTION.[18] The former is witnessed in (32), due to Krifka (1990), where 'is green and white' is a complex predicate.

(32) This flag is green and white.

Clearly, we would not conclude from (32) that this flag has the property of being both green all over and white all over. On Krifka's analysis, this follows from the semantic contribution of cumulative 'and', given in (33a).

(33) a. $[\![$and$]\!] = \lambda P, Q.\lambda x.\exists y, z.\ x = y \sqcup z \wedge P(y) \wedge Q(z)$
b. $[\![$green and white$]\!] = \lambda x.\exists y, z.\ x = y \sqcup z \wedge \mathtt{green}(y) \wedge \mathtt{white}(z)$

According to (33b), (32) is true if the thing demonstrated is a flag consisting of at least two parts: a green part and a white part. Because these two parts are clearly distinct, 'and' is naturally interpreted cumulatively: the green part and the white part *together* compose this flag.

Krifka also shows how his analysis naturally extends to attributive uses of 'green and white', such as (34a).

[18] See Partee and Rooth (1983), Krifka (1990), and Lasersohn (2013).

(34) a. This is a green and white flag.

b. $[\![\text{and}]\!] = \lambda\mathcal{P}_{<<e,t>,<e,t>>}.\mathcal{Q}_{<<e,t>,<e,t>>}\lambda P.\lambda x.\exists Q, Q'.\exists y, z.\forall x', x''.$ $[Q(x') \wedge Q'(x'') \rightarrow P(x' \sqcup x'')] \wedge x = y \sqcup z \wedge \mathcal{P}(Q)(y) \wedge \mathcal{Q}(Q')(z)$

c. $[\![\text{green and white}]\!] = \lambda P.\lambda x.\exists Q, Q'.\exists y, z.\forall x', x''.[Q(x') \wedge Q'(x'') \rightarrow P(x' \sqcup x'')] \wedge x = y \sqcup z \wedge \texttt{green}(Q)(y) \wedge \texttt{white}(Q')(z)$

d. $[\![\text{green and white flag}]\!] = \lambda x.\exists Q, Q'.\exists y, z.\forall x', x''.[Q(x') \wedge Q'(x'') \rightarrow \texttt{flag}(x' \sqcup x'')] \wedge x = y \sqcup z \wedge \texttt{green}(Q)(y) \wedge \texttt{white}(Q')(z)$

According to (34b), 'and' in (34a) conjoins two modifiers, namely 'green' and 'white', which both describe properties of a flag. Specifically, according to (34d), 'green and white flag' is a property of entities x which can be partitioned by properties Q and Q' of entities x' and x'' collectively describable as a flag, and x has as parts two such entities, the first of which is green and the second of which is white. The relevant partitioning properties here are presumably regions of a flag. Consequently, similar to (32), (34a) will be true if this is a flag which can be partitioned into green and white parts.

Following Krifka's analysis, S&B similarly analyze 'two and a/one half' on the II as a complex modifier, ultimately leading to the analysis in (35).[19]

(35) a. Those are two and a/one half oranges.

b. $[\![\text{two and one half}]\!] = \lambda P.\lambda x.\exists Q, Q'.\exists y, z.\forall x', x''.[Q(x') \wedge Q'(x'') \rightarrow P(x' \sqcup x'')] \wedge x = y \sqcup z \wedge \texttt{two}(Q)(y) \wedge \texttt{one-half}(Q')(z)$

c. $[\![\text{two and one half oranges}]\!] = \lambda x.\exists Q, Q'.\exists y, z.\forall x', x''.[Q(x') \wedge Q'(x'') \rightarrow \texttt{oranges}(x' \sqcup x'')] \wedge x = y \sqcup z \wedge \texttt{two}(Q)(y) \wedge \texttt{one-half}(Q')(z)$

Like (34a), 'and' in (35a) conjoins two modifiers, namely 'two' and 'one half'. These describe partitionings by properties of the things indicated into parts, which are collectively describable as oranges. However, the relevant partitioning properties in this case need to be different, due to the accompanying modifiers: the first is the plural property of being oranges, while the second is the singular property of being an orange.[20] As a result, (35a) will be true if the things deictically referenced can be partitioned into parts, the first of which is two oranges, the second of which is one half orange.

This presupposes an independent semantics for cardinal modifiers and fraction words. Beginning with the former, let's follow Scontras (2014) in assuming that the lexical meaning of number expressions is that of a numeral naming a number, as suggested in (36a).

[19] The question arises as to how cardinal '`one`' in (35) is contributed by 'a half'. S&B offer two tentative suggestions: (i) it could be that indefinites of the form 'a(n) *N*' are always synonymous with 'one *N*' (see Scontras (2014) for relevant references), or (ii) it could be that indefinites are synonymous with 'one *N*' when occurring as modifiers specifically in English (see Ionin and Matushansky (2006)'s discussion of "semi-lexical cardinals").

[20] This implies that at least with complex fractions, plural marking is not a reflection of the noun's meaning, but rather morphosyntactic features, possibly reflecting the conjunction. Compare 'one {orange/??oranges}' and 'one half {orange/??oranges} with 'one and one half {??orange/oranges}'. A similar view is found in Krifka (1989), who observes similar contrasts with 'zero {??orange/oranges}' and 'one point zero {??orange/oranges}'. See also Marti (2017) for discussion of these data.

(36) a. $[\![\text{two}]\!] = 2$

b. $[\![\text{CARD}]\!] = \lambda P.\lambda n.\lambda x.\, \#(x) = n \wedge P(x)$

c. $[\![\text{two CARD oranges}]\!] = \lambda x.\, \#(x) = 2 \wedge \texttt{oranges}(x)$

They can then come to function as cardinal modifiers by combining with the type-shifting operation CARD in (36b). As a result, 'two oranges' will denote sums of oranges having exactly two atomic parts.

As for fraction words, S&B follow Ionin et al. (2006) by analyzing simple fractions such as 'one half' as consisting of two parts: a denominator ('half') which "packages" entities into quantity uniform, non-overlapping parts, and a numerator ('one') which counts those parts. Specifically, the meaning of 'half' is given in (37), which I separate into three clauses for readability. According to the first, 'half' is a predicate modifier which divides the Ps into maximal parts. According to the second, these maximal parts are partitioned (Π) into two sets. According to third, these sets have the same measure, according to some contextually determined measure (μ).

(37) $[\![\text{half}]\!] = \lambda P.\lambda x.\exists y.$

i. $P(y) \wedge \forall z[P(z) \rightarrow z \sqsubseteq y] \wedge$

ii. $\exists S\, [\Pi(S)(y) \wedge |S| = 2 \wedge x \in S \wedge$

iii. $\exists \mu\, [\mu \in M \wedge \forall s_1, s_2\, [(s_1 \in S \wedge s_2 \in S) \rightarrow (\mu(s_1) = \mu(s_2))]]$

To illustrate, consider (38a).

(38) a. John ate one half of an orange.

b. John ate one half of the oranges.

By clause (i), 'an orange' has a maximal part, namely the whole orange. By clause (ii), that maximal part can be partitioned into two subparts which, by clause (iii), measure the same according to some contextually-determined measure, presumably volume. The resulting predicate can then combine with a cardinal modifier such as 'one', following (36). Accordingly, (38a) will be true if there is a partitioning of an orange into two halves, equal in volume, and John ate one of those halves. Similarly, (38b) will be true if there is a partitioning of a plurality of oranges into two halves, equal in cardinality, and John ate the oranges belonging to one of them.

This analysis of simple fractions can then be extended to complex fractions. As (35c) suggests, the latter are analyzed as having two components: a cardinal modifier such as 'two', which counts a plurality of objects denoted by the noun, and a simple fraction such as 'one half', which denotes counted parts in the way just suggested, both joined by cumulative 'and'. Since 'in the punch' is a distributive predicate, (10a) will be true on the II if there is a sum of things having two parts—a sum of two oranges, and a single half orange—each part of which is in the punch.

Returning now to COP, we saw above that it introduces an individuating context, and with it an II of 'two and a half oranges'. As a result, 'two and a half' is functioning as a complex modifier, one which is *counting* two oranges and one half orange. Hence, there are strictly speaking only two oranges on the table in that context, though there is also a half orange. Thus, unlike KIA, S&B's analysis is able to semantically distinguish the II from the MI, and correctly predicts that (7a) ought to receive an II in the context of COP.

What's more, COP presents no special threat to the Traditional View since (7a) does not ascribe a fractional cardinality on the II.

(7a) How many oranges are there on the table? $2\frac{1}{2}$.

But what about the *MI*? After all, '$2\frac{1}{2}$' remains a perfectly acceptable answer to the 'how many'-question in measure contexts like the one above. Yet, on S&B's analysis, the answer provided in (7a) involves *measuring* two and half oranges worth of orange in this context, where 'two and a half' functions as a numeral specifying the *amount* of orange-stuff on the table. Given Inquiry, this also specifies a *cardinality*. Thus, it would appear that fractional cardinalities are needed after all.

Here is our predicament. Initially, (7a) posed a challenge to the Traditional View because, assuming Inquiry, it appears to require postulating fractional cardinalities. Adopting KIA affords a coherent way of denying Discreteness, but has independent semantic problems. These problems can be overcome by adopting S&B's analysis. However, while this offers a nice defense of the Traditional View in cases involving IIs like COP, it offers no such defense in cases involving MIs, let alone cases involving measure phrases. Thus, FCP remains a genuine threat to the Traditional View.

The solution, I submit, is to deny that answers to 'how many'-questions always designate cardinalities. If Inquiry is false, then there is no guarantee that answers to questions like those in (7a,b) designate cardinalities, in which case such examples needn't motivate postulating *fractional* cardinalities. In the next section, I will argue that Inquiry should be rejected on independent grounds, thus making a better solution to FCP available.

3 Debunking inquiry

In this section, I consider and reject three arguments for Inquiry, each involving 'many'. Two come directly from Solt (2009), while the third is a natural extension of Solt's observations. Jointly, they purport to establish a thesis I call Uniform Dimensionality (UD):

> Uniform Dimensionality: 'Many', unlike 'much', is associated with the dimension of cardinality across all contexts.

According to Solt, 'many' and 'much' are both gradable adjectives, modeled as tuples $\langle D, \leq, \Delta \rangle$, where D is a set of degrees, $\leq$ is an ordering on D, and Δ is a dimension of measurement along which members of D are ordered.[21] The crucial contention is that 'many' and 'much' differ precisely in this third parameter: whereas 'much' can associate with different dimensions of measurement in different contexts, e.g. volume or weight, 'many' encodes cardinality as part of its lexical meaning, and so is associated with the dimension of cardinality across contexts. If so, then 'how many'-questions uniformly inquire into cardinalities, thus vindicating Inquiry.

[21] See also Rett (2008, 2014) and Portner (2009).

3.1 Three arguments for uniform dimensionality

The first argument for UD actually involves 'how many'-questions. Specifically, Solt reports the following contrast.

Context: John buys five 1 lb. oranges.

(39) a. Mary: "How many oranges did you buy?"
b. John: "{Five/??Five pounds}."

Whereas cardinal 'five' provides an acceptable answer to Mary's question, the measure phrase 'five pounds' apparently does not, despite the latter accurately describing the collective weight of the oranges Mary bought. Apparently, Mary's question is about the *cardinality* of the oranges, not their collective weight. This makes sense, of course, if 'how many'-questions always ask about the cardinality of a collection, just as UD predicts.

The second argument for UD involves cardinality comparisons like (40a,b).

Context: John buys five 2 lb. oranges. Mary buys five 1 lb. oranges.

(40) a. Mary bought as many oranges as John.
b. Mary bought as much orange as John.

Whereas (40a) is clearly true in the scenario described, (40b) is not. Intuitively, that's because while the *number* of oranges John and Mary bought is the same, the *amount* of orange John bought exceeds the amount Mary bought. As before, this makes sense if the equative 'as many as' always compares the cardinality of two collections, while 'as much as' always compares the amount of two substances according to some other salient dimension. And, again, this is exactly what UD predicts.

The final argument for UD involves anaphorically referenced cardinalities. As Scontras (2014) observes, 'amount' is ambiguous between counting and measuring interpretations. Thus, consider (41a), which is ambiguous between (41b) and (41c).

Context: Pointing at five 1 lb. oranges

(41) a. John ate that amount of oranges every day for a year.
b. Every day for a year, John ate exactly five oranges, possibly differing in their collective weight.
c. Every day for a year, John ate oranges collectively weighing five pounds, possibly differing in their cardinality.

As Scontras explains, (41b) arises if 'amount' is interpreted as measuring the cardinality of the oranges, while (41c) arises if it is instead interpreted as measuring their weight. In either case, 'that amount' anaphorically references a degree—a degree of cardinality or a degree of weight—instantiated by the oranges John ate.

Building on Scontras' observation, Snyder (2017) points out that 'number' disambiguates 'amount' by selecting for a *counting* interpretation. For example, in the same context, (42) is naturally interpreted as (41b).

(42) John ate that number of oranges every day for a year.

According to Snyder, that's because 'number' relates pluralities to degrees of cardinality, as a matter of its lexical meaning. With this in mind, consider (43), which is also naturally interpreted as (41b).

(43) John ate that many oranges every day for a year.

As with 'number', this makes sense if 'many' specifically relates pluralities to degrees of cardinality, as a matter of its lexical meaning. If so, then we would appear to have a direct argument for UD.

Based on these kinds of observations, Solt (2009, p. 63) reasonably concludes that 'many' encodes cardinality, and that this is not merely a reflection of morphological agreement, contra Chierchia (2005).

> If the presence of *many*/*few* versus *much*/*little* were determined simply by morphological agreement with a plural noun, then we would expect that their interpretation could reference any dimension on which the given plurality could be appropriately measured. But the examples in [(39, 40)] show that this is not the case; rather, *many*/*few* can only reference cardinality. The conclusion must be that *many* and *few*, and the complex expressions formed from them, encode cardinality.

That's because, as Solt (2014, p. 12) later explains, unlike 'much', 'many' *lexically* encodes the dimension of cardinality.

> I thus propose that *many*/*few* lexically encode cardinality as a dimension of measurement. *Much*/*little* I take to be unspecified for dimension of measurement, with the result that they may be used in a wider range of contexts; the exception is with canonical plural count nouns, where they are blocked by the more specific *many*/*few*.

This, in turn, vindicates UD, and with it, Inquiry: since 'many' lexically encodes cardinality, 'how many'-questions always inquire into cardinalities.

3.2 Debunking uniform dimensionality

We have seen three arguments for UD. This section debunks those arguments by presenting novel evidence which, relying on similar assumptions, shows that 'many' cannot *uniformly* encode cardinality. Rather, like 'much', 'many' can encode different dimensions in different contexts.

Let's begin by revisiting 'how many'-questions. The claim was that unlike cardinal modifiers, measure phrases are generally unacceptable in response to 'how many'-questions, and that this makes sense only if 'many' lexically encodes cardinality. There are two problems with this argument, however. First, it is not obvious whether measure phrases used in response to 'how many'-questions are *always* unacceptable. For example, contrast the acceptability of John's responses in the different contexts provided.

Context 1: Mary is decorating the table. John knows that Mary wants to match each apple with an orange, but she is unsure whether John has bought enough oranges to do this.

(44) a. Mary: "How many oranges did you buy?"
b. John: "{Five (oranges)/??Five pounds (of oranges)}.

Context 2: Mary is making punch for the party. John knows that Mary needs five pounds of orange pulp to complete the recipe, but she is unsure whether John has purchased enough oranges to do this.

(45) a. Mary: "How many oranges did you buy?"
b. John: "{Five (oranges)/Five pounds (of oranges)}.

This suggests that the apparent unacceptability of measure phrases in response to 'how many'-questions is plausibly a matter of pragmatics, not semantics. If so, then (44b) offers no support for UD.

The second problem is that similar examples can be produced which directly undermine UD. For example, consider a parallel example involving 'glasses of water', repeated in (12a), which is clearly ambiguous between an II paraphrased in (12b) and a MI in (12c).

I-Context: Same as (12). John asks:

M-Context: Same as (12). John asks:

(12a) How many glasses of water did you put in the soup?

(12b) How many glasses filled with water did you put in the soup?

(12c) How many glassfuls of water did you put in the soup?

We saw above that in the I-Context, 'glasses of water' receives an II, paraphraseable as "glasses filled with water". As a result, cardinals combining with 'glasses of water' in that context *count* individuated glasses. Similarly, (12a) is naturally interpreted as asking about the cardinality of those glasses in that context, thus leading to (12b). In contrast, in the M-Context, 'glasses of water' receives a MI, and so *measures* an amount of water according to an ad hoc glass-unit. Similarly, (12a) is naturally interpreted as inquiring into the *volume* of water Mary put into the soup, measured in terms of glassfuls, thus leading to (12c). Clearly, this ambiguity is not expected given UD since, in that case, (12a) would only have an II.

Next reconsider cardinality comparisons. The claim was that 'as many oranges as' can only be interpreted as comparing the cardinality of two collections of oranges, since 'many' lexically encodes cardinality. But now consider a parallel example involving container phrases.

I-Context: Similar to (12). Whereas Mary sets five normal sized glasses filled with water in her soup, John sets ten small glasses filled with water in his soup, exactly half the size of Mary's.

M-Context: Similar to (12). Mary makes soup by pouring water directly from five normal sized glasses into her soup. John does the same, but with ten small glasses, exactly half the size of Mary's.

(46) Mary put as many glasses of water in the soup as John.

Clearly, (46) can receive different evaluations in these contexts. It is naturally interpreted as false in the I-Context, since the cardinality of Mary's glasses is less than that of John's. On the other hand, it is naturally interpreted as true in the M-Context, since Mary and John poured the same amount of water into their respective soups. Again, such divergences are unexpected given UD, as (46) would only have a false interpretation.

Finally, consider again anaphoric reference. The claim was 'that many oranges' anaphorically references a cardinality, since 'many', like 'number', relates pluralities to degrees of cardinality, as a matter of its meaning. However, consider a parallel example involving 'that many glasses of water'.

I-Context: Similar to (12). John sets five normal sized glasses filled with water into his soup. Mary does the same, though her glasses are only half the size of John's.

M-Context: Similar to (12). John fills a normal sized glass five times with water, pouring the contents each time directly into his soup. Mary does the same, though her glass is only half the size of John's.

(47) a. John: "I put five glasses of water in my soup. How about you?"
b. Mary: "I put that many glasses of water in my soup too."

Again, we see a difference in possible evaluations here. In the I-Context, (47b) is naturally interpreted as true, as John and Mary set the same number of glasses in their respective soups. In contrast, (47b) can be interpreted as false in the M-Context, as the amount of water Mary put in her soup is only half of what John put in his. As before, this is unexpected given UD, as it predicts that (47b) should only have a true interpretation.

I have used container phrases to debunk UD because they are exemplars of I/M-ambiguous expressions. However, it should be emphasized that the same point could be made using 'oranges' instead. Witness (48).

I-Context: Mary and John are decorating punch for the party. Mary drops five normal sized oranges into her punch. John drops ten small oranges in his punch, exactly half the size of Mary's.

M-Context: Mary and John are making punch, which calls for five oranges worth of orange pulp. Accordingly, Mary pulverizes five normal sized oranges, pouring the resulting pulp into her punch. John does the same with ten small oranges, exactly half the size of Mary's.

(48) Mary put as many oranges in the punch as John.

Again, (48) can receive different evaluations in these contexts. It can be interpreted as false in the I-Context, as the number of oranges John dropped into his punch is twice the number Mary dropped into hers. On the other hand, it can be interpreted as true in the M-Context, as John and Mary put the same amount of orange pulp into their respective punches.

The emerging generalization is that while ‘many’ assumes cardinality in some contexts, it needn’t do so in *every* context. This suggests that ‘many’ does not *lexically* encode cardinality. Rather, like ‘much’, ‘many’ plausibly associates with different dimensions of measurement in different contexts, including volume. If so, then there is no guarantee that (7a) on the MI or (7b) are inquiring into *cardinalities*.

(7a) How many oranges are there on the table? $2\frac{1}{2}$.

(7b) How many ounces of water are in the beaker? 2.38.

If instead ‘many’ encodes *volume* in these contexts, as seems reasonable, then the answers to these questions will also specify volumes. Consequently, (7a,b) would pose no special threat to the Traditional View, nor would they provide independent motivation for UDM.

Of course, it remains to be seen whether a semantics can be formulated for examples like (7a,b) which is actually consistent with the Traditional View. This is the goal of the next section. I will show how combining a modified version of Rett (2008)’s semantics for ‘many’ with independently motivated analyses of degrees, measure phrases, and complex fractions predicts intuitively correct truth-conditions for (7a,b) and similar examples. The upshot is that fractional cardinalities are not required in either case.

4 Salvaging the traditional view

Before moving on to develop a new semantics in defense of the Traditional View, let me briefly summarize the dialectic up to this point. We began by noting that (7a,b) appear problematic for the Traditional View, assuming that answers to ‘how many’-questions uniformly specify cardinalities.

(7a) How many oranges are there on the table? $2\frac{1}{2}$.

(7b) How many ounces of water are in the beaker? 2.38.

That’s because fractional cardinalities are incoherent by the lights of the Traditional View, and yet these are precisely what the answers provided in (7a,b) apparently designate. A similar point can be made using (49b) and (50b), where ‘that many oranges’ and ‘that many ounces of water’ appear to anaphorically reference cardinalities.

(49) a. John: “I put $2\frac{1}{2}$ oranges in my punch. How about you?”
b. Mary: “I put that many oranges in my punch too.”

(50) a. John: “I put 2.38 ounces of water in my beaker.”
b. Mary: “I put that many ounces of water in my beaker too.”

We then saw that while adopting a hybrid Krifka-Rett analysis would make sense of these fractional cardinalities, it had its own empirical shortcomings. Finally, we saw that there are good, independent reasons for rejecting Inquiry, and with it the assumption that the expressions just mentioned must designate cardinalities.

The plan of this section is to defend the Traditional View against the threat of these examples by developing a semantics on which (7a,b), as well as (49b) and (50b), receive intuitively correct truth-conditions *without* postulating fractional cardinalities. More specifically, the plan is to improve upon Rett (2008)'s original analysis of 'many', by dropping three of her original assumptions. The first is that 'many' lexically encodes cardinality, and so uniformly maps degrees to a cardinality scale. Instead, I allow that 'many' can map degrees to different scales, including discrete cardinality scales and dense volume scales, depending on context. Secondly, I drop the assumption that the degrees measured by 'many' are generated via QUANTITY. As we saw in Sect. 2, the MI of '$2\frac{1}{2}$ oranges' plausibly results from coercing 'orange' into a reflexivized measure noun, one which measures an amount of orange relative to an ad hoc orange unit. This cannot be captured via QUANTITY, however, assuming count nouns always induce cardinality measures. Happily, there are other independently motivated type-shifting principles which can generate sets of degrees appropriate for combining with 'many', as we will soon see. However, this alternative will work only if we also drop the assumption that degrees are real numbers.[22]

Instead, I adopt Scontras (2014)'s analysis, according to which degrees are NOMINALIZED PROPERTIES, or KINDS in the sense of Chierchia (1998). More specifically, degrees are nominalized properties of measured substances. Schematically, degrees take the form in (51), where '$^{\cap}$' is Chierchia's nominalization operator mapping predicates to kinds qua names of the corresponding properties, and 'μ_f' is a measure function mapping entities to real numbers according to some unit of measurement f.

(51) $^{\cap}[\lambda x.\ \mu_f(x) = n \wedge P(x)]$

To illustrate, consider (52a,b).

(52) a. $^{\cap}[\lambda x.\exists n.\ \mu_{\#}(x) = n \wedge \texttt{oranges}(x) \wedge \texttt{on-table}(x)]$
b. $^{\cap}[\lambda x.\exists n.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(x) \wedge \texttt{in-beaker}(x)]$

(52a) is a degree of *cardinality*: it names a property of sums of oranges on the table having a certain number of atomic parts. In contrast, (52b) is a degree of *volume*: it names a property of quantities of water in the beaker measuring a certain amount, in ounces. Since degrees are built from an independent domain of real numbers, they cannot be non-circularly identified with those numbers. Thus, Scontras' primary motivation for instead characterizing degrees as kinds comes from numerous semantic parallels between degree-denoting and kind-denoting expressions. However, a similar conception is defended by Anderson and Morzycki (2015), based on different considerations. Call it THE DEGREE-AS-KIND ANALYSIS (DKA).

One advantage of adopting DKA, which is central to the analysis developed below, is that it affords a way of generating degrees which can compositionally combine with

[22] This identification has also been recently challenged on independent grounds. See e.g. Moltmann (2013), Scontras (2014), Anderson and Morzycki (2015), Balcerak-Jackson and Penka (2017) and Snyder (2017).

'many', without assuming QUANTITY. I will illustrate with the II of 'oranges'. First, assume Scontras' analysis of cardinal modification from Sect. 2.2, repeated here in (36a–c).

(36a) $[\![\text{two}]\!] = 2$

(36b) $[\![\text{CARD}]\!] = \lambda P.\lambda n.\lambda x.\ \#(x) = n \wedge P(x)$

(36c) $[\![\text{two CARD oranges}]\!] = \lambda x.\ \#(x) = 2 \wedge \texttt{oranges}(x)$

Thus, number expressions come to function as cardinal modifiers in virtue of combining with the type-shifting operation CARD. In the absence of overt numerals, however, we may follow Krifka (1989) in assuming that numerical arguments get existentially bound, as is presumably witnessed in e.g. 'many oranges', thus leading to (53).[23]

(53) $[\![\varnothing \text{ CARD oranges}]\!] = \lambda x.\exists n.\ \mu_{\#}(x) = n \wedge \texttt{oranges}(x)$

We now assume Landman (2004)'s ADJUNCT-shifter in (54a), which adjoins two predicates.

(54) a. $\text{ADJUNCT} = \lambda P.\lambda Q.\lambda x.\ P(x) \wedge Q(x)$
b. $\text{ADJUNCT}([\![\varnothing \text{ CARD oranges}]\!])([\![\text{on the table}]\!]) = \lambda x.\exists n.\ \mu_{\#}(x) = n \wedge \texttt{oranges}(x) \wedge \texttt{on-table}(x)$

Thus, adjoining the predicate in (53) with 'on the table' leads to a complex predicate true of sums of oranges which are on the table and have a certain number of atomic parts.[24] Finally, we generate a degree of cardinality (henceforth 'cardinality') from (54b) by applying two of Partee (1986)'s type-shifters, namely NOM, which codifies Chierchia's $^{\cap}$-operator, and IDENT, which generates an identity predicate from a singular term.[25]

(55) a. $\text{NOM} = \lambda P_{<\alpha,t>}.\ ^{\cap}[\lambda x_{<\alpha>}.\ P(x)]$
b. $\text{IDENT} = \lambda x_{<\alpha>}.\lambda y_{<\alpha>}.\ y = x$

Applying NOM to (54b) returns the nominalized property of being oranges on the table having a certain number of atomic parts, i.e. a cardinality.

(56) $\text{NOM}([\![(54b)]\!]) = {}^{\cap}[\lambda x.\exists n.\ \mu_{\#}(x) = n \wedge \texttt{oranges}(x) \wedge \texttt{on-table}(x)]$

Applying IDENT to the result thus returns a set of cardinalities identical to the one generated in (56).

(57) $\text{IDENT}[\![(56)]\!]) = \lambda d.\ d = {}^{\cap}[\lambda x.\exists n.\ \mu_{\#}(x) = n \wedge \texttt{oranges}(x) \wedge \texttt{on-table}(x)]$

23 See also Rett (2014).

24 This exploits the fact that cardinal modifiers are intersective. See Landman (2004), Rothstein (2017), and Snyder (2017).

25 Following Kennedy (2015), I use 'α' to range over sorts of individuals. Also, following Chierchia (1998) and Scontras (2014), I assume kinds are individuals.

Hence, we have replicated the effect of QUANTITY without identifying degrees with real numbers. As we will see, a similar process will generate degrees appropriate for the MI of 'oranges', as well as measure phrases.

Having generated degrees, we can then order them into scales. The crucial difference will be that whereas the resulting cardinality scales are discrete, measurement scales are dense.

$$\begin{array}{cc} \textsc{Cardinality Scales} & \textsc{Measurement Scales} \\ \left\{\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ {}^{\cap}[\lambda x.\ \mu_{\#}(x) = 2] \\ {}^{\cap}[\lambda x.\ \mu_{\#}(x) = 1] \end{array}\right\} & \left\{\begin{array}{c} \cdot \\ {}^{\cap}[\lambda x.\exists n.\ \mu_{vol,oz}(x) = 2] \\ \cdot \\ {}^{\cap}[\lambda x.\exists n.\ \mu_{vol,oz}(x) = 1] \\ \cdot \end{array}\right\} \end{array}$$

In either case, we get a set of degrees which ℓ can then measure, just as with Rett's original analysis.[26]

(13) $[\![\text{many/much}]\!] = \lambda D_{<d,t>}.\lambda d.\ \ell[D] = d$

In what follows, I will show how combining this modified version of Rett's analysis with other independently motivated semantic resources predicts intuitively correct truth-conditions for examples like (7a,b), (49), and (50).

The resulting semantics vindicates the Traditional View by coherently maintaining the difference in scale structures pictured above, while predicting intuitively correct truth-conditions for the otherwise problematic examples mentioned. In other words, it provides appropriate meanings for those examples without needing to postulate fractional cardinalities. For reasons which will become evident, I begin with measure phrases.

4.1 That many ounces of water

As in Sect. 1, I assume that measure phrases denote an amount of a substance measured in terms of a canonical unit named by the noun.

(2a) $[\![\text{ounce}]\!] = \lambda P.\lambda n.\lambda x.\ \mu_{oz,vol}(x) = n \wedge P(x)$

(2c) $[\![\text{two ounces of water}]\!] = \lambda x.\ \mu_{oz,vol}(x) = 2 \wedge \texttt{water}(x)$

Given DKA, it is easy to see how we can generate a degree of volume from (2c), as would be appropriate for '2.38 ounces of water' in (58): we simply nominalize the predicate.

(58) How many ounces of water are in the beaker? 2.38 ounces of water.

[26] This assumes a kind of structuralism: because degrees of measurement are isomorphic to real numbers, and since degrees of cardinality form a substructure of degrees of measurement, Rett's measurement theoretic ℓ-operation is defined for both.

However, in order to generate a set of degrees appropriate for combining with ‘many’, we continue following Krifka (1989) in assuming that covert numerical arguments get existentially bound, as suggested in (59).

(59) $[\![\varnothing \text{ ounces of water}]\!] = \lambda x.\exists n.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(x)$

Adjoining the resulting predicate with ‘in the beaker’ and applying NOM and IDENT to the result thus generates a set of degrees of *volume*.[27]

(60) a. $\text{NOM}([\![\varnothing \text{ ounces of water in the beaker}]\!]) = {}^{\cap}[\lambda x.\exists n.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(x) \wedge \texttt{in-beaker}]$

b. $\text{IDENT}([\![(60a)]\!]) = \lambda d.\ d = {}^{\cap}[\lambda x.\exists n.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(x) \wedge \texttt{in-beaker}(x)]$

Hence, we now have a denotation appropriate for combining with ‘many’. Doing so gives us (61).

(61) $[\![\text{many}]\!]([\![(60b)]\!]) = \lambda d'.\ \ell[\lambda d.\ d = {}^{\cap}[\lambda x.\exists n.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(x) \wedge \texttt{in-beaker}(x)]] = d'$

Assuming that ‘many’ maps the degrees in (60b) to a volume scale, we can then form a ‘how many’-question inquiring into the volume of water in the beaker, given Rett (2008)’s analysis from above.

(62) $\lambda p.\exists d'[p(w_@) \wedge p = \lambda w.\ \ell[\lambda d.\ d = {}^{\cap}[\lambda x.\exists n.\ \mu_{oz,vol}(x) = n \wedge \texttt{water}(w)(x) \wedge \texttt{in-beaker}(w)(x)]] = d']$

As a result, it is hardly surprising that the correct answer to that question specifies a degree which does not belong to a discrete set.

Turning now to (50b), the question becomes: How can we combine the demonstrative ‘that’ with ‘many ounces of water’ to anaphorically reference a degree of volume?

(50a) John: “I put 2.38 ounces of water in my beaker.”

(50b) Mary: “I put that many ounces of water in my beaker too.”

To this end, I borrow Scontras (2014)’s analysis given in (63), where ‘$^{\cup}$’ is Chierchia’s “up”-operator mapping a kind to its instances in a given world, and ‘THAT’ refers to some salient entity demonstrated in context.

(63) $[\![\text{that}]\!] = \lambda P.\ \iota x_{<\alpha>}[P(x) \wedge {}^{\cup}x(\texttt{THAT})]$

Intuitively, ‘that many ounces of water’ in (50b) refers to a degree of volume instantiated by the water in John’s beaker. This is achieved by combining (63) with the denotation for ‘many ounces of water’ in (61).[28]

[27] This assumes that measure phrases, like cardinal modifiers, are intersective. See Schwarzschild (2005).

[28] This requires viewing ι as a maximality-operation (Chierchia 1998) which selects the maximum ounces-of-water-degree true of THAT.

(64) $[\![\text{that}]\!]([\![(61)]\!]) = \iota d[\texttt{many-ounces-of-water}(d) \wedge {}^{\cup}d(\texttt{THAT})]$

Here, '$\texttt{THAT}$' references the water in John's beaker, and 'many ounces of water' denotes a set of degrees of volume instantiated by quantities of water. Thus, 'that many ounces of water' will refer to the maximal such degree instantiated by the water in John's beaker. We then apply Chierchia (1998)'s DERIVED KIND PREDICATION (DKP), which takes a kind k and returns a second-order property of things instantiating k and satisfying P.[29]

(65) DKP $= \lambda k.\lambda P.\exists x.\, {}^{\cup}k(x) \wedge P(x)$

Applying DKP to (64) results in truth-conditions for (50b) given in (66b).

(66) a. $\text{DKP}([\![(64)]\!]) = \lambda P.\exists x.\, {}^{\cup}[\iota d[\texttt{many-ounces-of-water}(d) \wedge {}^{\cup}d(\texttt{THAT})]](x) \wedge P(x)$
b. $\exists x.\, {}^{\cup}[\iota d[\texttt{many-ounces-of-water}(d) \wedge {}^{\cup}d(\texttt{THAT})]](x) \wedge \texttt{Mary-put-in-beaker}(x)$

In English, (50b) will be true if Mary put a quantity of water in her beaker instantiating the same degree of volume as the water in John's beaker. That is, it will be true if Mary and John put the same amount of water in their respective beakers.

Crucially, the degree anaphorically referenced here is not a *cardinality*, but a volume, despite the presence of 'many'. Consequently, there is no need to postulate fractional cardinalities to explain the truth of (7b) or (50b).

4.2 That many oranges

We have seen how to derive a meaning appropriate for '2.38 ounces of water' consistent with the Traditional View. Providing a similar analysis for (7a) is less straightforward, however, due to I/M-ambiguities.

(7a) How many oranges are there on the table? $2\frac{1}{2}$.

What we want is an analysis of the MI on which '$2\frac{1}{2}$' specifies the volume of the orange-stuff on the table, and an analysis of the II on which '$2\frac{1}{2}$' specifies the cardinalities of the oranges and half-oranges on the table. I will show how to do this in what follows, beginning with the MI.

4.2.1 Measure interpretations

We begin by adopting Snyder and Barlew (2019)'s analysis, whereby '$2\frac{1}{2}$ oranges' in (49a) denotes a measure of orange-stuff relative to an ad hoc orange-unit, presumably given in terms of volume.

(31b) $[\![2\frac{1}{2} \text{ oranges}]\!] = \lambda x.\, \gamma(\texttt{oranges})(x) \wedge \mu_{orange,vol}(x) = 2.5$

[29] On this characterization, DKP is an operator, although in Chierchia (1998) it is a general type-shifting mechanism.

As before, it is easy to see how we can generate a degree of volume from this predicate given DKA: we nominalize it.

(67) $^{\cap}[\lambda x.\ \gamma(\texttt{oranges})(x) \wedge \mu_{orange,vol}(x) = 2.5]$

Intuitively, on the MI, 'that many oranges' in (49b) anaphorically references this degree. And given the preceding analysis of 'that many ounces of water', it is easy to see how. First, we apply RUM to 'oranges', thus returning a measure of orange-stuff.

(68) $\text{RUM}([\![\text{oranges}]\!]) = \lambda n.\lambda x.\ \gamma(\texttt{oranges})(x) \wedge \mu_{orange,vol}(x) = n$

Existentially binding the covert numerical argument and applying NOM and IDENT to the result returns a set of degrees of volume instantiated by quantities of orange.

(69) $\lambda d.\ d = {}^{\cap}[\lambda x.\exists n.\ \gamma(\texttt{oranges})(x) \wedge \mu_{orange,vol}(x) = n]$

Assuming 'many' is associated with the dimension of volume in this context, the 'how many'-question in (7a) will ask about the volume of orange-stuff in the punch relative to an ad hoc orange-unit, the correct answer to which is given by the degree in (67).

(70) $\lambda p.\exists d'[p(w_{@}) \wedge p = \lambda w.\ \ell[\lambda d.\ d = {}^{\cap}[\lambda x.\exists n.\ \gamma(\texttt{oranges})(w)(x) \wedge \mu_{orange,vol}(x) = n \wedge \texttt{in-punch}(w)(x)]] = d'\ ]$

In order to anaphorically reference that degree, we apply 'many', 'that', and DKP to (69), ultimately resulting in truth-conditions for (49b) given in (71).

(71) $\exists x.{}^{\cup}[\iota d[\texttt{many-oranges}(d) \wedge {}^{\cup}d(\texttt{THAT})]](x) \wedge \texttt{Mary-put-in-punch}(x)$

In English, (49b) will be true on the MI if there is a quantity of orange in Mary's punch instantiating the same degree of volume as the quantity of orange demonstrated in John's punch. In other words, it will be true if both punches contain the same amount of orange-stuff. Thus, as with examples involving measure phrases, there is no need to postulate fractional cardinalities to explain the truth of (7a) on the MI or (49b), as once again the relevant degrees referenced are volumes, not cardinalities.

However, now consider a slight variation on (49).

Context: John and Mary are making punch. They both begin with five oranges, though Mary's oranges are exactly half the size of John's. They pulverize their oranges, pouring the resulting orange pulp into their respective punches.

(72) a. John: "I put five oranges in my punch. How about you?"
b. Mary: "I put that many oranges in my punch too."

Intuitively, (72b) can be interpreted as true in the measure context provided. Yet if measure contexts always induce MIs, and if MIs always result from measuring quantities of orange-stuff relative to a typical whole orange, then (72b) should be true just in case John and Mary put the same volume of orange-stuff in their respective punches, contrary to fact.

The simplest solution, it seems, would be to abandon the assumption that measure contexts always induce measure interpretations. If we instead allow that 'many' can be associated with the dimension of cardinality in such contexts, so that ‘that many oranges’ anaphorically references the cardinality of John’s oranges prior to pulverizing them, then (72b) would be true simply in virtue of John’s and Mary’s oranges sharing a cardinality, pre-pulverization. Further motivation for abandoning this assumption comes from (73), which as Susan Rothstein (p.c.) observes, can also be interpreted as true in response to (72a).

(73) Mary: “I put the same number of oranges in my punch.”

Assuming with Snyder (2017) that ‘number’ encodes, or at least strongly prefers, cardinality, this would make sense if ‘that many oranges’ and ‘the same number of oranges’ reference the same degree of cardinality, despite both utterances taking place within a measure context.[30]

4.2.2 Individuating interpretations

Turning now to IIs, recall that on Snyder and Barlew (2019)’s analysis, ‘$2\frac{1}{2}$ oranges’ involves counting two whole oranges and one half orange. An immediate consequence is that Mary’s response in (49b) should be interpretable as true on the II even if she failed to include a half orange in her punch.

(49a) John: “I put $2\frac{1}{2}$ oranges in my punch. How about you?”

(49b) Mary: “I put that many oranges in my punch too.”

Admittedly, this interpretation is not easy for everyone to get.[31] However, evidence for the prediction comes from the fact that (49b) can followed up with ‘... though I didn’t put a half orange in my punch’ without contradiction, along with variations on examples like (74).

Context 1: Mary and John are participating in a bobbing for oranges contest. The barrel contains numerous whole oranges and half oranges. Points are awarded as follows: whole oranges are worth 3 points, while half oranges are worth 1. In total, Mary bobs for five half oranges, while John bobs for two whole oranges and one half orange.

Context 2: Exactly like Context 1, except Mary bobs for two whole oranges.

(74) a. John: “I bobbed for $2\frac{1}{2}$ oranges. How did you do?”
b. Mary: “We tied, since I also bobbed for that many oranges.”

30 Alternatively, one could hold that (72b) is true in virtue of John’s oranges and Mary’s oranges instantiating the same *ratio* of orange-stuff to an ad hoc orange-unit, where different ad hoc orange-units are operative in this context (one of John’s oranges vs. one of Mary’s oranges), recalling Partee and Borschev (2012)’s observation that ad hoc units are essentially context-dependent. However, it is not clear whether this suggestion can explain contrasts like (46) in Sect. 3, which is naturally interpreted as true on the MI despite different ratios obtaining between the ad hoc units employed.

31 According to an anonymous reviewer, it is much easier to interpret (49b) as suggesting that in fact Mary put a half orange in her punch. One seemingly plausible possibility, consistent with the semantics developed below, is that this is due to ellipsis, so that (49b) is effectively interpreted as ‘I put that many oranges ~~and half oranges~~ in my punch too’.

Given the scoring system, Mary's utterance is clearly false in both contexts provided. However, if the scoring system were changed, so that half oranges count for no points, then Mary's utterance would be true in the second context but not the first, intuitively.

In this respect, (75) differs importantly from (49b), also uttered in response to (49a).

(75) Mary: "I put that many oranges and half oranges in my punch too."

Unlike (49b), (75) would be false if Mary failed to put a half orange in her punch. Thus, the question becomes: How can we generate cardinalities referenced by 'that many oranges' and 'that many oranges and half oranges' appropriate for these different responses?

We begin by assuming that sets of cardinalities are generated via the same process mentioned above, using CARD, ADJUNCT, NOM, and IDENT, ultimately resulting in (76).

(76) $\lambda d.\, d = {}^{\cap}[\lambda x.\exists n.\ \mu_{\#}(x) = n \wedge \texttt{oranges}(x) \wedge \texttt{in-punch}(x)]$

This can then combine with 'many', thus returning a measured set of discrete cardinalities.

(77) $[\![\text{many}]\!]([\![(76)]\!]) = \lambda d'.\ \ell[d = {}^{\cap}[\lambda x.\exists n.\ \mu_{\#}(x) = n \wedge \texttt{oranges}(x) \wedge \texttt{in-punch}(x)]] = d'$

From here, we can form 'how many'-questions asking about the cardinality of the atomic oranges in the punch. Alternatively, we can form a set of cardinalities from 'many oranges', combining the result with anaphoric 'that' and DKP, ultimately leading to truth-conditions for (49b) on the II.

(78) $\exists x.\ {}^{\cup}[\iota d[\texttt{many-oranges}(x) \wedge {}^{\cup}d(\texttt{THAT})]](x) \wedge \texttt{in-punch}(x)$

In English, (49b) will be true if there are oranges in Mary's punch instantiating the same degree of cardinality as those demonstrated in John's. In other words, it will be true if there are two whole oranges in Mary's punch.

Things are less straightforward with (75), however. The trouble is that because the oranges and the half orange have different cardinalities, it is not immediately obvious how we can generate cardinalities from 'oranges and half oranges' appropriate for combining with 'many'. To see how to proceed, it will help to consider (79a).

(79) a. I put two apples and oranges in the punch.
b. I put two apples and two oranges in the punch.
c. I put one apple and one orange in the punch.

Clearly, (79a) can mean (79b) but not (79c). Put differently, 'two' "distributes" over 'apples and oranges'.[32] Following Schmitt (2013), I assume this is due to ellipsis of the modifier. Consequently, we can apply Krifka (1990)'s analysis of cumulative predicate conjunction from Sect. 2.2. The result is that (79a) and (79b) are equivalent, as suggested in (80).

[32] As a reviewer observes, it is also possible to interpret (79a) non-distributively, as 'two apples and *some* oranges'. My sense is that in that case, speakers would normally use 'some' explicitly so as to prevent the salient distributive interpretation.

(80) $[\![\text{two apples and } \cancel{\text{two}} \text{ oranges}]\!] = \lambda x.\exists y, z.\ x = y \sqcup z \wedge \texttt{two(apples)}(y) \wedge \texttt{two(oranges)}(z)$

Accordingly, (79a) will be true just in case there is a sum of things on the table consisting of two parts: (i) a sum of two apples, and (ii) a sum of two oranges, each part of which is in the punch.

Next, notice that 'many' similarly "distributes" over 'apples and oranges'. Thus, in response to (79a), (81) can mean (79b) but not (79c).

(81) I put that many apples and oranges in the punch too.

As before, I assume this is due to ellipsis. Unlike 'two', however, 'many' maps sets of degrees to sets of degrees. Nevertheless, because degrees are *kinds* under present assumptions, and since kinds are sorts of individuals (Chierchia 1998), they can be joined together by the sum-operation in the same way "ordinary" individuals like John and Mary can (Scontras 2014). Consequently, given Krifka's analysis, 'and' in (81) joins degrees in the manner suggested in (82).

(82) $[\![\text{many apples and } \cancel{\text{many}} \text{ oranges}]\!] = \lambda d.\exists d', d''.\ d = d' \sqcup d'' \wedge \texttt{many(apples)}(d') \wedge \texttt{many(oranges)}(d'')$

Generating sets of cardinalities from 'apples' and 'oranges' in the same manner described above, we then apply generalized cumulative conjunction to the result, giving us (82), or a sum of cardinalities collectively instantiated by sums of apples and oranges.

This can then combine with 'that' and DKP, ultimately resulting in the truth-conditions suggested in (83b).

(83) a. $[\![\text{that}]\!]([\![(82)]\!]) = \iota d[\exists d', d''.\ d = d' \sqcup d'' \wedge \texttt{many(apples)}(d') \wedge \texttt{many(oranges)}(d'') \wedge {}^{\cup}d(\texttt{THAT})]$

b. $\exists x.\ {}^{\cup}[\iota d[\texttt{many-apples-and-oranges}(d) \wedge {}^{\cup}d(\texttt{THAT})]](x) \wedge \texttt{in-punch}(x)$

According to (83a), 'that many apples and oranges' refers to a sum of cardinalities, $d' \sqcup d''$, respectively instantiated by the demonstrated sums of apples and oranges. Since those sums are both two in number, d will be collectively instantiated by sums having two parts: (i) sums of two apples, and (ii) sums of two oranges. Applying DKP results in the following truth-conditions: (81) is true if Mary put apples and oranges in the punch having the same respective cardinalities as those anaphorically referenced, i.e. if she put two apples and two oranges in the punch.

Truth-conditions for (75) can be similarly derived, with one important difference: we are counting oranges and half oranges having different cardinalities. First, we generate cardinalities from 'oranges' and 'half oranges', and sum these via generalized conjunction.

(84) $[\![\text{many oranges and } \cancel{\text{many}} \text{ half oranges}]\!] = \lambda d.\exists d', d''.\ d = d' \sqcup d'' \wedge \texttt{many(oranges)}(d') \wedge \texttt{many(half-oranges)}(d'')$

As before, this denotes a sum of cardinalities collectively instantiated by oranges and half oranges. Combining this with anaphoric 'that' and DKP then returns truth-conditions for (75) in (85b).

(85) a. $[\![\text{that}]\!]([\![(84)]\!]) = \iota d[\exists d', d''. d = d' \sqcup d'' \wedge \texttt{many(oranges)}(d') \wedge \texttt{many(half-oranges)}(d'') \wedge {}^{\cup}d(\texttt{THAT})]$

b. $\exists x.{}^{\cup}[\iota d[\texttt{many-oranges-and-half-oranges}(d) \wedge {}^{\cup}d(\texttt{THAT})]](x) \wedge \texttt{in-punch}(x)$

Accordingly, 'that many oranges and half oranges' anaphorically references a sum of cardinalities jointly instantiated by the demonstrated oranges and half oranges. Since there are two demonstrated oranges and one demonstrated half orange, (75) will be true if there is a plurality consisting of two parts, (i) a plurality of two oranges and (ii) one half orange, each part of which Mary put in the punch. In other words, it will be true if Mary put two whole oranges and a single half orange in the punch.

To summarize, as with Snyder and Barlew (2019)'s original solution to the Counting Oranges Puzzle, the semantics developed here suggests that there is no need to postulate fractional cardinalities in order to solve the Fractional Cardinalities Puzzle. With measure phrases and MIs of '$2\frac{1}{2}$ oranges', 'many' measures degrees of volume, not cardinality. And on IIs of '$2\frac{1}{2}$ oranges', 'many' does measure degrees of cardinality, but these are only instantiated by pluralities of atoms. Hence, neither kind of purported counterexample to the Traditional View is in fact a counterexample. We only need to recognize that 'many', like 'much', can take on different dimensions of measurement in different contexts, as revealed in Sect. 3.

5 Conclusion

I have argued that despite initial appearances, examples such as (7a,b) do not require postulating fractional cardinalities, and so lend no independent support for the view that all natural language scales are dense.

(7a) How many oranges are there on the table? $2\frac{1}{2}$.

(7b) How many ounces of water are in the beaker? 2.38.

Rather, fractional cardinalities are needed only if an independently problematic assumption is adopted, namely that answers to 'how many'-questions always designate cardinalities. This plausibly follows if 'many' lexically encodes cardinality. However, examples involving I/M-ambiguous container phrases reveal that 'many' can be associated with different dimensions of measurement in different contexts. Thus, solving the Fractional Cardinalities Puzzle does not require abandoning the assumption that cardinality scales are discrete, but rather the assumption that answers to 'how many'-questions uniformly designate cardinalities.

In defending the Traditional View, I have shown how meanings appropriate for (7a,b) naturally arise from combining independently motivated analyses of cardinal modifiers, measure phrases, fractions, and 'many'. First, adopting Snyder and Barlew

(2019)'s analysis of individuating/measure ambiguities accounts for how mixed fractions like '$2\frac{1}{2}$' in (7a) can, but need not, involve measuring quantities of a substance. Secondly, adopting Scontras (2014)'s analysis of degrees as kinds affords a natural way of generating various kinds of degrees, including degrees of cardinality and volume, from both (complex) cardinal phrases and measure phrases. Finally, combining these with a modified version of Rett (2008)'s analysis of 'many', which can now associate with different dimensions of measurement in different contexts, results in intuitively correct truth-conditions for (7a,b) and similar, otherwise problematic examples.

A crucial part of my argument turns on the contention that 'many' is like 'much' in that both can associate with different dimensions of measurement in different contexts, thus leading to different kinds of operative measures: cardinality and volume. Several follow up questions naturally arise, of course. For example, how exactly do 'many' and 'much' select for different measures? Which, if any, constraints are in place? If there are any constraints, what do these reflect—differences in the lexical meanings of different adjectives, nouns they combine with, or something else?

Promising answers to these questions are suggested in recent work on comparatives and similar constructions by Wellwood (2018, 2019). First, consider that equatives with 'many' and 'much' both naturally select for different kinds of measures with different kinds of nouns. For example, 'containers of water' in (86a) naturally selects for cardinality, while 'ounces of water' naturally selects for volume. Similarly, 'furniture' in (86b) naturally selects for cardinality,[33] whereas 'water' naturally selects for volume.

(86) a. Mary has as many {containers/ounces} of water as John.
 b. Mary has as much {furniture/water} as John.

Minimally, this shows that 'much' too can select for cardinality. Thus, it seems that 'many' and 'much' can select for the same sorts of measures.

On the other hand, Schwarzschild (2002) has shown that measure selection is not completely unconstrained. For example, (86b) with 'water' would not be true if Mary's water happens to be higher in temperature than John's, despite being less in volume. Schwarzschild thus imposes a constraint on acceptable measures known as MONOTONICITY, which roughly requires that measure functions track part-whole relations. Cardinality and volume are monotonic in this sense: adding atomic oranges to a sum of oranges increases its cardinality, and vice versa, just as adding water to a puddle of water increases its volume, and vice versa. In contrast, temperature is not monotonic: adding water to a puddle needn't make it hotter, for instance.

Thus, monotonicity constrains measure selection by excluding non-monotonic measures. But as Wellwood observes, it alone will not explain why comparatives with plural nouns typically select for cardinality. For example, (87) naturally selects for cardinality, even though volume is also monotonic.

(87) Mary has more coffees than John.

Thus, Wellwood proposes another constraint on measure selection dubbed A(UTOMORPHISM)- INVARIANCE. Roughly, this requires that measures be strongly

[33] See Barner and Snedeker (2005) and Bale and Barner (2009).

structure preserving in the sense that any automorphic mapping from a domain onto itself preserves the original ordering relations on entities within that domain. In atomic domains, the only A-invariant measure will be cardinality, as only it preserves original parthood relations between atoms and sums. In non-atomic domains, however, cardinality is unavailable, as there are no atoms to count. Thus, the only A-invariant measures available for such domains will be monotonic measures like volume and weight.

Wellwood's claim is that monotonicity and A-invariance jointly suffice to explain measure selection. For this to work, however, it needs to be that different domains are differently structured, and that comparatives and similar constructions are sensitive to those differences. To this end, Wellwood argues for two further claims. First, 'many' and 'much' are ALLOMORPHS: different phonological realizations of the same morpheme. Specifically, 'much' realizes a lexical primitive 'MUCH', while 'many' realizes 'MUCH' combined with the plural morpheme 'PL'.[34] The semantic effect of the former is to select for measure functions subject to monotonicity and A-invariance, while the semantic effect of the latter is to form sums from atoms. As a result, there is no need to posit distinct lexemes corresponding to 'many' and 'much'. A fortiori, differences in measure selection cannot be due to differences in the lexical meanings of different adjectives.

Secondly, the comparative 'more' likewise decomposes into 'MUCH' combined with the morpheme 'ER'. Measure selection for comparatives like (87) is then explained by the domain structures of the accompanying noun. Prototypical mass nouns like 'coffee' denote non-atomic substances, and so only select for measures such as volume and weight. On the other hand, pluralizing 'coffee' imposes an atomic structure on its domain, thus making cardinality the only selectable measure, at least in the typical case.[35] A similar explanation is proposed for so-called OBJECT MASS NOUNS like 'furniture', supposing they too denote within atomic structures.[36] Thus, that we can pluralize e.g. 'coffee' but not 'furniture' ultimately reflects morphosyntax, not meaning. A fortiori, the contrast {many coffees/??many furnitures} is also a reflection of morphosyntax.

In sum, measure selection is determined by the structure of the domains assumed, in conjunction with monotonicity and A-invariance. As a result, atomic domains are measured by cardinality, while non-atomic domains require other monotonic measures. More evidence for this conclusion comes from psycholinguistic experiments like that

[34] See Wellwood (2018, 2019) for cross-linguistic evidence.

[35] The analysis developed in Sect. 2.2 predicts that measure interpretations ought to be available even for prototypical cases of "packaging", such as with 'five beers' (Pelletier 1975). (i) Supports this prediction, as (ib) is true in C1 but not C2, intuitively:

(C1) Mary and John are making soup. The recipe calls for 5 cans of beer. Mary pours 5 normal sized cans of beer into her soup, while John pours 10 small cans of beer into his, exactly half the size of Mary's.

(C2) Just like C1, only John pours 5 small beers into his soup.

(i) a. Mary: "I put five beers in my soup, per the recipe. How about you? Did you follow the recipe?"
b. John: "Yes. I put just as many beers in my soup."

[36] See e.g. Barner and Snedeker (2005), Bale and Barner (2009), Grim and Levin (2016) and Rothstein (2017).

of Barner and Snedeker (2005),[37] along with more recent experiments conducted by Scontras et al. (2017, p. 13), who summarize their results as follows:

> Confronted with linguistically-cued atomic nominals (i.e. plural count or partitioned mass nouns), participants perform quantity judgments by counting the named atoms; for linguistically-cued non-atomic nominals (i.e. bare mass and singular count), participants perform quantity judgments by measuring the volume of the relevant substances. Thus, the primary driver of behavior in the quantity judgment task is the quantity judgment prompt itself. When the prompt fails to provide the information necessary to determine atomicity ... participants default to the ontological category of the visualized material (i.e., atomic individuals vs. non-atomic substance).

The upshot for the present analysis is that while individuating interpretations assume atomic domains, measure interpretations instead assume non-atomic domains, in virtue of meaning coercion. As such, cardinality or volume are both selectable for 'that many oranges', but in a principled, constrained manner. In contrast, 'that many ounces of water' will only select for volume, given the non-atomic nature of the substance measured.

I conclude that, despite initial appearances, examples like (7a,b) provide no evidence against the view that natural language distinguishes counting from other forms of measurement. Ultimately, while this does not represent a *complete* defense of the Traditional View—that would require addressing Fox and Hackl (2007)'s original arguments for UDM—it does answer what I take to be its most direct and intuitively compelling empirical challenge.

References

Anderson, C., & Morzycki, M. (2015). Degrees as kinds. *Natural Language & Linguistic Theory*, *33*(3), 791–828.

Balcerak-Jackson, B., & Penka, D. (2017). Number word constructions, degree semantics and the metaphysics of degrees. *Linguistics and Philosophy*, *40*(4), 347–372.

Bale, A. (2008). A universal scale of comparison. *Linguistics and Philosophy*, *31*(1), 1–55.

Bale, A., & Barner, D. (2009). The interpretation of functional heads: Using comparatives to explore the mass/count distinction. *Journal of Semantics*, *26*, 217–252.

Barner, D., & Snedeker, J. (2005). Quantity judgments and individuation: Evidence that mass nouns count. *Cognition*, *97*(1), 41–66.

Barwise, J., & Cooper, R. (1981). Generalized quantifiers and natural language. *Linguistics and Philosophy*, *4*(2), 159–219.

Breheny, R. (2008). A new look at the semantics and pragmatics of numerically quantified noun phrases. *Journal of Semantics*, *25*(2), 93–139.

Carrara, M., & Lando, G. (2017). Composition and relative counting. *Dialectica*, *71*(4), 489–529.

Chierchia, G. (1998). Reference to kinds across languages. *Natural Language Semantics*, *6*, 339–405.

Chierchia, G. (2005). Mass nouns, number marking and semantic variation. Talk given at CUNY Linguistics Colloquium, November 10, 2005.

Fox, D., & Hackl, M. (2007). The universal density of measurement. *Linguistics and Philosophy*, *29*(5), 537–586.

Frege, G. (1884). *Grundlagen der Arithmetik*. Berlin: Springer.

[37] See also Bale and Barner (2009).

Frege, G. (1903). *Grundgesetze der Arithmetik II*. Hildesheim: Olms.
Grim, S., & Levin, B. (2016). Artifact nouns: Reference and countability. In A. Lamont, & K. Tetzloff (Eds.), *Proceedings of the 47th Annual Meeting of North East Linguistic Society (NELS 47)* (Vol. 2, pp. 55–64). Amherst, MA: GLSA.
Groenendijk, J., & Stokhof, M. (1991). Dynamic predicate logic. *Linguistics and Philosophy*, *14*(1), 39–100.
Hackl, M. (2000). *Comparative quantifiers*. PhD thesis, Massachusetts Institute of Technology.
Ionin, T., & Matushansky, O. (2006). The composition of complex cardinals. *Journal of Semantics*, *23*(4), 315–360.
Ionin, T., Matushansky, O., & Ruys, E. G. (2006). Parts of speech: Toward a unified semantics for partitives. In C. Davis, A. R. Deal, & Y. Zabbal (Eds.), *Proceedings of the 36th Annual Meeting of the North East Linguistic Society (NELS 36)* (Vol. 1, pp. 357–370). Amherst MA: GLSA.
Karttunen, L. (1977). Syntax and semantics of questions. *Linguistics and Philosophy*, *1*(1), 3–44.
Kennedy, C. (2015). A “de-fregean” semantics (and neo-gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics*, *8*(10), 1–44.
Kennedy, C., & Stanley, J. (2009). On average. *Mind*, *118*, 586–646.
Krifka, M. (1989). Nominal reference, temporal constitution, and quantification in event semantics. In J. von Bentham, R. Bartsch, & P. von Emde Boas (Eds.), *Semantics and contextual expressions* (pp. 72–115). Dordrecht: Foris.
Krifka, M. (1990). Boolean and non-Boolean *and*. In L. Kalman & L. Polos (Eds.), *Papers from the Second Symposium on Logic and Language* (pp. 161–188). Budapest: Akademmiai Kiado.
Landman, F. (2004). *Indefinites and the type of sets*. Oxford: Blackwell.
Lasersohn, P. (2013). *Plurality, conjunction and events*. Berlin: Springer.
Liebesman, D. (2015). We do not count by identity. *Australasian Journal of Philosophy*, *93*(1), 21–42.
Liebesman, D. (2016). Counting as a type of measuring. *Philosopher’s Imprint*, *16*, 1–25.
Link, G. (1983). The logical analysis of plurals and mass terms: A lattice-theoretic approach. In R. Bäuerle, C. Schwarze, & A. von Stechow (Eds.), *Meaning, use, and interpretation of language* (pp. 303–323). Berlin: De Gruyter.
Marti, L. (2017). *Zero N: Number features and* $\perp$. Ms., Queen Mary University of London.
Moltmann, F. (2013). Reference to numbers in natural language. *Philosophical Studies*, *162*, 499–536.
Nicolas, D. (2016). Interprétons-nous de la même manière les expressions deux pommes et deux pommes et demie? *Travaux de linguistique*, *1*, 107–119.
Partee, B. (1986). Noun phrase interpretation and type-shifting principles. In J. Groenendijk, D. de Jongh, & M. Stokhof (Eds.), *Studies in discourse representation theory and the theory of generalized quantifiers* (pp. 114–144). Dordrecht: Foris.
Partee, B., & Borschev, V. (2012). Sortal, relational, and functional interpretations of nouns and russian container constructions. *Journal of Semantics*, *29*, 445–486.
Partee, B., & Rooth, M. (1983). Generalized conjunction and type ambiguity. In R. Bäuerle, C. Schwarze, & A. von Stechow (Eds.), *Meaning, use, and interpretation of language* (pp. 361–383). Berlin: De Gruyter.
Pelletier, F. J. (1975). Non-singular reference: Some preliminaries. In F. J. Pelletier (Ed.), *Mass terms: Some philosophical problems* (pp. 1–14). Dordrecht: Springer.
Portner, P. (2009). *Modality*. Oxford: Oxford University Press.
Rett, J. (2008). *Degree modification in natural language*. PhD thesis, Rutgers University.
Rett, J. (2014). The polysemy of measurement. *Lingua*, *143*, 242–266.
Rothstein, S. (2009). Individuating and measure readings of classifier constructions: Evidence from modern Hebrew. *Brill’s Annual of Afroasiatic Languages and Linguistics*, *1*, 106–145.
Rothstein, S. (2016). Counting and measuring: A theoretical and crosslinguistic account. *Baltic International Yearbook of Cognition, Logic and Communication, 11*. https://doi.org/10.4148/1944-3676.1106.
Rothstein, S. (2017). *Semantics for counting and measuring*. Cambridge: Cambridge University Press.
Rullman, H. (1993). Scope ambiguities in *how many*-questions. Ms., University of Massachusetts.
Salmon, N. (1997). Wholes, parts, and numbers. *Nous Supplement: Philosophical Perspectives 11: Mind, Causation, and World*, *31*, 1–15.
Schmitt, V. (2013). Boolean and non-Boolean conjunction. Ms., Universität Wien.
Schwarzschild, R. (2002). The grammar of measurement. *Semantics and Linguistic Theory*, *12*, 225–245. https://doi.org/10.3765/salt.v12i0.2870.
Schwarzschild, R. (2005). Measure phrases as modifiers of adjectives. *Recherches Linguistiques de Vincennes*, *34*, 207–228.

Scontras, G. (2014). *The semantics of measurement*. PhD thesis, Harvard University.
Scontras, G., Davidson, K., Deal, A. R., & Murray, S. E. (2017). Who has more? The influence of linguistic form on quantity judgments. *Proceedings of the Linguistic Society of America, 2*. https://doi.org/10.3765/plsa.v2i0.4097.
Snyder, E. (2017). Numbers and cardinalities: What's really wrong with the easy argument? *Linguistics and Philosophy*, *40*(4), 373–400.
Snyder, E., & Barlew, J. (2016). The universal measurer. *Proceedings of Sinn und Bedeutung, 20*, 658–675. https://ojs.ub.unikonstanz.de/sub/index.php/sub/article/view/288.
Snyder, E., & Barlew, J. (2019). How to count $2\frac{1}{2}$ oranges. *Australasian Journal of Philosophy*, *97*, 792–808.
Snyder, E., & Shapiro, S. (2016). Frege on the reals. In P. Ebert & M. Rossberg (Eds.), *Essays on Frege's basic laws of arithmetic* (pp. 343–383). Oxford: Oxford University Press.
Solt, S. (2009). *The semantics of adjectives of quantity*. PhD thesis, The City University of New York.
Solt, S. (2014). Q-adjectives and the semantics of quantity. *Journal of Semantics*, *32*(2), 221–273.
Wellwood, A. (2018). Structure preservation in comparatives. *Semantics and Linguistic Theory*, *28*, 78–99. https://doi.org/10.3765/salt.v28i0.4413.
Wellwood, A. (2019). WHAT more IS. Philosophical Perspectives. *Perspectives*, *32*(1), 454–486.