MATHEMATICS HANDBOOK

(For students who joined in 2020 or later)

ashoka
UNIVERSITY

YEAR 2021-22

Some of the most powerful and beautiful ideas occur in the field of Mathematics. The wide applicability of these ideas and their deep connection with the natural sciences have made this discipline one of the most fruitful arenas of human inquiry. Combining, as it does, the greatest creative freedom with the most stringent standards of rigor, Mathematics also happens to be the ideal training ground for learning a broad range of analytical and problem-solving skills. Ashoka University's Mathematics major program has been designed to meet two primary goals:

1. Students should get a broad exposure to the primary areas and the central ideas of contemporary Mathematics (as well as their applications) and
2. Students should develop rigorous, analytical reasoning skills, along with problem-solving ability.

In addition to the Foundation courses that are common across disciplines, students aiming to choose Mathematics as their major should take the enabling course on Calculus as early as possible. This will be followed by **11** *required* and **3** *elective* courses in Mathematics. The required courses cover the areas of Analysis, Algebra, Linear Algebra, Probability, and Differential Equations. Elective courses vary from semester to semester depending on student and faculty interests. In addition, students have the opportunity to study some special topics in depth under the guidance of a faculty member.

At the end of the program of study, we expect students to be able to read and understand mathematical proofs; learn and apply new mathematical concepts; and, construct and communicate a correct and rigorous argument on their own. Most importantly, we expect students to be able to solve *new* mathematical problems on their own. Students completing this program will be well prepared to pursue Mathematics further or to take up positions that call for innovative problem solving in concert with strong analytical abilities.

Course Requirements

B.Sc. (Hons) with Major in Mathematics

Requirements: Each student will take a total of 15 mathematics courses. For completing a major in Mathematics, one must take the following courses (12 required courses and 3 elective courses).

Required Courses

1000 Level Courses

- **Calculus**
  - This course will be offered every semester.
- **Linear Algebra**
- **Multivariable Calculus**
  - This course is expected to be offered in every even semester
  - Prerequisite: Calculus

2000 Level Courses

The following are expected to be offered in odd semesters

- **Algebra 1**
- **Probability and Statistics**
- **Real Analysis**
  - Prerequisite: Calculus

The following are expected to be offered in even semesters

- **Algebra 2**
  - Prerequisite: Algebra 1
- **Metric and Topological Spaces**
  - Prerequisites: Calculus, Real Analysis

3000 Level Courses

The following are expected to be offered in odd semesters

- **Complex Analysis**
  - Prerequisites: Calculus, Multivariable Calculus, Real Analysis, Linear Algebra
- **Mathematical Modeling (Differential Equations)**
  - Prerequisites: Calculus, Multivariable Calculus, Real Analysis, Linear Algebra
- **Linear Algebra and Matrix Analysis**
  - Prerequisites: Calculus, Linear Algebra, Real Analysis

The following is expected to be offered in even semesters

- **Elementary Differential Geometry**

    - Prerequisites: Calculus, Multivariable Calculus, Linear Algebra, Real Analysis, Metric and Topological Spaces

Elective Courses

In addition to the 12 required courses, three electives are required. The elective courses offered vary from semester to semester depending on student interest and the availability of faculty. The elective courses offered by the department so far are as follows.

CTS Course

- **Introduction to Proofs**
    - This cannot be counted as a math elective if counted as a CTS requirement.

3000 Level Courses

- **Statistical inference I**
- **Fourier Analysis**
- **Introduction to Combinatorics**
- **Galois' dream**
- **Introduction to Algebraic Combinatorics**

4000 Level Courses

- **Topological Spaces**
- **Measure Theory**
- **Functional Analysis**
- **Random Graphs**
- **Topics in Analysis**
- **Mathematical Foundations of Data Sciences**
- **Algebraic Number Theory**
- **Topics in Stochastic Processes**

B.Sc. (Hons) with Major in Mathematics and Computer Science

Requirements: *Each student will take a total of 9 Mathematics courses and 9 Computer Science Courses and will top it up with any course either from Mathematics / Computer Science avoiding double counting. The academic requirement for this interdisciplinary major is given below:*

Required Courses

Mathematics Department (Credits: 36)

- **Calculus**
- **Linear Algebra**
- **Algebra I**
- **Probability and Statistics**
- **Real Analysis**
- **Multivariable Calculus**
- **Statistical Inference I**
- ***Two more mathematics courses (8 credits) have to be taken***

Computer Science Department (Credits: 36)

- **Introduction to Computer Programming**
- **Computer Organization and Systems**
- **Algorithm Design and Analysis**
- **Computer Networks**
- **Introduction to Machine Learning**
- **Computer Security and Privacy**
- **Theory of Computation**
- ***Two more computer science courses (8 credits) have to be taken***

In addition to the above, one more CS or Math course (double counting not allowed) has to be taken (4 credits).

Minor in Mathematics

Required Courses

1. Calculus

2. Multivariable Calculus

3. Linear Algebra

4. Algebra I

5. Probability and Statistics

6. Real Analysis

Students taking a course in probability as part of their major are required to replace the Probability course with another math course of their choice.

Concentration in Mathematics

Required Courses

1. Calculus

2. Linear Algebra

3. Algebra I

One more course must also be taken, of the student's choice. Note that this course ***cannot*** *be Probability theory if they have done a course on Probability in their major.*

Suggested Course Progression

A student majoring in math should take Calculus in the first semester. This course is a prerequisite to further mathematics courses. Students thinking of studying mathematics should take this course as early as possible, possibly the first or the second semester. A student who wants to decide later can take the Math FC in the first semester. If a student is interested in the mathematics major, the student should follow the suggested course sequence for math majors starting from the second semester. The following tables show the intended course sequence for math majors (for students who joined in 2019 or later).

For Students taking Calculus in the First Semester (2019 or later)

Semester 1	Semester 2	Semester 3	Semester 4	Semester 5	Semester 6
*Calculus**#	Linear Algebra *#	Algebra I *#	Algebra II	Complex Analysis	Differential Geometry
	Multivar. Calculus*#	Probability *	Met. & Top. Spaces	Math Modeling (Diff. eqns.)	Elective +
					Elective +
		Analysis *		Linear Algebra & Matrix Anal.	Elective +
				Elective +	

Please see Notes below for markings.

For Students taking Calculus in the Second Semester (2019 or later)

Semester 1	Semester 2	Semester 3	Semester 4	Semester 5	Semester 6
	Linear Algebra *#	Algebra I *#	Algebra II	Complex Analysis	Differential Geometry
	*Calculus**#	Probability *	Met. & Top. Spaces	Math Modeling (Diff. eqns.)	Elective +
					Elective +
		Analysis *	Multivar. Calculus #	Linear Algebra & Matrix Anal.	Elective +
				Elective +	

Notes

Calculus is a mandatory course for all Mathematics majors and minors.

Students are required to take a minimum of **3** Elective + courses preferably in their 5^{th} and 6^{th} semester.

Courses **marked*** are **mandatory** for Mathematics minors.

For a Concentration in Mathematics one needs to do courses **marked#** and **one more** course of their choice, provided pre-requisites are met.

Frequently Asked Questions (FAQs)

Q: Is the 1000/2000/3000 level system an equivalent of the 100/200/300 level system?

A: Yes.

Q: Is it mandatory for majors to take calculus by the end of first year?

A: Yes. Otherwise it would be very difficult to complete your math major in 3 years. Indeed, it is extremely helpful to take calculus by the end of first year in case you intend to major in other mathematically-oriented majors such as Physics, Computer Science and Economics.

Q: Is Multivariable Calculus the same course as Calculus II?

A: Yes.

Q: Is 'Probability and Statistics' course the same course as Probability Theory?

A: Yes, it will be treated as the same course. Students who joined in 2019 or earlier are required to take the Probability Theory course and students who joined in 2020 (or later) need to take the Probability and Statistics course to complete a math major.

Q: Now the Probability and Statistics course is offered, will the Statistical Inference course be offered?

A: Yes, it will be offered as an elective course; the course content for Statistical Inference is different from Probability and Statistics.

Q: Is it mandatory for majors to take Multivariable Calculus by the end of second year?

A: No, it is not mandatory. But it would be helpful while doing Real Analysis. But if you cannot do it in the first year, you can still do it in your 4th semester.

Q: Is it mandatory for majors to take Linear Algebra by the end of first year?

A: Yes, it is necessary to complete a math major in 3 years.

Q: What is the policy on cross-listed courses?

A: Cross-listed courses are those courses which arise out of a discipline different from Mathematics but are cross-listed with mathematics. The students can take cross-listed courses towards their Major. The information of cross-listed courses (if there are any) will be shared with the students.

Q: I took a course in Monsoon 2019 which was cross-listed with CS. It is being offered again in 2020 but is not showing as cross-listed. Is this a problem?

A: No. It is normal for courses to not be cross-listed with other departments in every semester that they're offered. If you took a course, which was cross-listed in the semester that you took it, it will count towards your degree. Please cross-check the course code of such courses in your LMS.

Q: Can I write a thesis in the third year?

A: No. Only ASPs with prior approval from the department can write a capstone thesis with the department.

Q: I'm going on a semester abroad. Can I substitute the required courses with a summer abroad course?

A: The specific course along with its detailed syllabus will need to be shared with the Department's HOD and a decision will be taken on a case-by-case basis.

Q: I'm interested in TA-ing for the Monsoon semester. How should I proceed?

A: Only 3rd year and 4th year students are eligible to TA for courses. The Department notifies the students about TA requirements in the beginning of the semester and calls for applications.

Q: Whom should I contact for further queries?

A: You can email the student representative Aishani Pradhan (aishani.pradhan_ug22@ashoka.edu.in) and cc the Department Manager Tejasvi Anand (tejasvi.anand@ashoka.edu.in) for any queries. For course related matters, you can also email Professor Kumarjit Saha (kumarjit.saha@ashoka.edu.in).

Q: I am confused about which elective courses to opt for. Is there someone I can talk to?

A: You may consult the course descriptions provided on the LMS or may reach out to your peers or seniors who have already taken courses you may be interested in. You may also seek guidance from the respective instructor.

Q: Is there an order that courses have to be taken in?

A: Yes. The course progression section provides a recommended order, which takes into account the prerequisites of each course.

Q: Where can I find more information related to course descriptions and syllabus?

A: You can consult the website https://ashoka.edu.in/mathematicsdepartment and under the tab 'Programs' click on 'Math Major'. Each semester's courses will have descriptions on LMS as well. If you have further queries, consult one of the contacts above.

Course Outlines

The following are brief descriptions of the mandatory courses. The contents of the actual courses and references followed may be different.

MAT 1000: Calculus

Syllabus: Number systems. Sequences and series. Functions of a real variable. Graphs of functions. Limits and continuity. Differentiation. Mean value theorem. L'Hospital rule. Maclaurin and Taylor series. Curve tracing. Riemann integral. Definite and indefinite integrals. Fundamental theorem of calculus. Applications of differential and integral calculus in areas such as optimization and mechanics.

References:

1. J. Stewart: Calculus, Cengage Publishers, 2012.
2. K. A. Ross: Elementary Analysis, The Theory of Calculus, Second Edition, Undergraduate Texts in Mathematics, Springer, 2013.
3. G. B. Thomas and R. L. Finney: Calculus and Analytic Geometry, Second Edition, Addison-Wesley Publishing, 1998.

MAT 1001: Linear Algebra

Syllabus: Real vector spaces, subspaces, spanning sets, basis sets, dimension of a vector space. Solution of a system of linear equations. Row space and column space of a matrix, rank of a matrix, elementary row and column operations of a matrix. Inversion of square matrices, rank factorization of a matrix. Properties of determinants. Linear transformations, range and null space of a linear transformation, rank-nullity theorem. Matrix representation of a linear transformation. Inner product spaces, normed linear spaces, examples of different normed linear spaces, orthonormal basis sets. Eigenvalues, eigenvectors, characteristic polynomials. Spectral theorem for real symmetric matrices. Singular value decompositions.

References:

1. A. R. Rao and P. Bhimsankaram: Linear algebra, Hindustan book agency, 2000.
2. S. H. Friedberg, A. J. Insel and L. E. Spence: Linear algebra, Pearson, 2015.
3. D. C Lay: Linear algebra and its applications, Pearson, 2014.

MAT 1004: Multivariable Calculus

Syllabus: Review of vectors and matrices. Curves and surfaces. Partial derivatives. Total differential and gradients. Maximum and minimum values. Lagrange multipliers. Double integrals, Fubini's theorem. Line integrals in the plane. Green's theorem. Triple integrals and surface integrals in 3-space. Stokes' theorem. Applications of multivariable calculus.

Prerequisite(s): Calculus

References:

1. James Stewart: Calculus, Cengage Publishers, 2012.
2. J.E. Marsden and A.J. Tromba: Vector Calculus, W. H. Freeman, 2003.
3. S. Lang: Calculus of several variables, Springer, 3rd edition, 1996.

MAT 2001: Algebra 1

Syllabus

Groups: Group structure and examples. Subgroups and cosets. Normal subgroups and Quotient groups. Lagrange, Euler and Fermat's theorem. Homomorphism, Isomorphism, Automorphism. Group actions. Class equation, Cauchy's theorem, Cayley's theorem. Simplicity of alternating groups. Sylow theorems.

Rings: Rings, Integral domains and fields. Isomorphism, homomorphism and quotient fields. Ideals - prime and maximal. Euclidean domain, division rule. Polynomials, irreducibility and Eisenstein's criterion. Chinese remainder theorem.

References:

1. M. Artin: Algebra, Second Edition, Pearson Prentice-Hall of India, New Delhi, 2011.

2. D. S. Dummit and Richard M. Foote: Abstract Algebra, Third Edition, Wiley, 2005.

3. J. A. Gallian: Contemporary Abstract Algebra, Eighth Edition, BROOKS/COLE Cengage Learning, 2013

4. Yvette Kosmann-Schwarzbach: Groups and Symmetries: From Finite Groups to Lie Groups by Springer, 2010.

5. I. S. Luthar and I. B. S. Passi. Algebra Vols I & 2, Narosa, 1996, 1999.

MAT 2002: Algebra 2

Syllabus:

Rings and Fields: U.F.D., P.I.D, factorization of polynomials. Field extensions. Normal extensions, Separable extensions. Galois extensions, Galois group. Fundamental theorem of Galois Theory. Cyclic Extensions, Solvability by radicals. Geometric constructions.

Groups: Solvable and nilpotent groups. Presentation of groups. Fundamental theorem for finitely generated Abelian groups. Semi-direct products, amalgamated products and HNN- extensions.

Prerequisite(s): Algebra 1.

References:

1. M. Artin: Algebra, Second Edition, Pearson Prentice-Hall of India, New Delhi, 2011.

2. D. S. Dummit and Richard M. Foote: Abstract Algebra, Third Edition, Wiley, 2005.

3. Patrick Morandi: Field and Galois Theory. Springer, 1996.

4. Yvette Kosmann-Schwarzbach: Groups and Symmetries: From Finite Groups to Lie Groups. Springer, 2010.

5. I. S. Luthar and I. B. S. Passi: Algebra Vols I & 2, Narosa, 1996, 1999.

MAT 2003: Real Analysis

Syllabus: Real and complex number systems. Limits of sequences. Monotonic sequences. Limits superior and limits inferior. Convergence of a series. Absolute and conditional convergence. Power series over real and complex numbers and their radius of convergence. Bolzano-Weierstrass Theorem, Cantor and Heine-Borel Theorems. Point wise and uniform continuity. Sequences and series of functions. Point wise and uniform convergence of sequence of functions. Integrals and derivatives of sequences and series of functions. Elementary transcendental functions. Improper integrals, Riemann-Stieltjes integral. Idea of Lebesgue integral, Weierstrass approximation Theorem, Inverse function theorem. Implicit function theorem.

Prerequisite(s): Calculus.

Desirable: Multivariable Calculus.

References:

1. T. M. Apostol: Mathematical Analysis, Second Edition, Addison-Wesley Publishing Company, 1974.
2. T. Tao: Analysis I, Hindustan Book Agency, 2017.
3. T. Tao: Analysis II, Hindustan Book Agency, 2017.
4. K. A. Ross: Elementary analysis: the theory of calculus, Springer, 2nd edition, 2013.

MAT 2020: Probability and Statistics

Syllabus: Frequency and axiomatic definition of probability, random experiments with equally likely finite outcomes, Inclusion exclusion principle. General finite sample spaces, infinite sample spaces. Concept of probability spaces and construction of probability measures. Conditional probability, Bayes theorem, Independence of events. Random variable (discrete), probability mass function and distribution function. Examples: Bernoulli, Binomial, Poisson, Geometric distributions. Expectation and variance of a random variable, sum law and product law of expectation, moment generating functions. Random vector (discrete), joint distribution, Marginal distributions, joint moment generating functions, covariance, Multinomial distributions. Continuous random variables, density functions, distribution functions, expectation, variance, moment generating function, example: uniform, normal, and exponential. Continuous random vector, joint density function, joint distribution function, conditional density, example: multivariate normal.

Inequalities: Markov, Chebyshev. Weak variant of law of large numbers, Central Limit Theorem (without proof).

Descriptive statistics, Distribution of sampling statistics, Parameter Estimation and Hypothesis testing basics.

Simple linear regression with one regressor (only if time permits).

References:

1. S. M. Ross: First Course in Probability, Pearson.
2. S. M. Ross: Introduction to Probability and Statistics for Engineers and Scientists
3. J. L. Devore: Probability and Statistics for Engineering, Cengage, 8th edition, 2012.
4. V. K. Rohatgi, E. S. Saleh: An Introduction to Probability and Statistics, Wiley-Blackwell, 3rd edition, 2015.

Previously, MAT 2020 was offered as MAT 2006 (Probability Theory). Students who have already taken MAT 2006 cannot register for MAT 2020.

MAT 2026: Metric and Topological Spaces

Syllabus: Metric spaces, open and closed sets. Euclidean spaces, normed linear spaces, examples of different normed linear spaces, sequence spaces. Completeness, Baire category Theorem. Compactness, characterization of compact spaces. Product spaces, Tychonoff's theorem. Continuous functions, equicontinuous families, Arzela-Ascoli Theorem. Connectedness, path connectedness.

General topological spaces, separation axioms. Hausdorff spaces. Convergence of nets.

Prerequisites: Calculus, Real Analysis, Linear algebra.

Desirable: Multivariable Calculus.

References:

1. M. O. Searcoid: Metric Spaces, Springer, 2007.
2. G. F. Simmons: Introduction to topology and modern analysis, Krieger Publishing, 2003.
3. S. Shirali and H. L. Vasudeva: Metric Spaces, Springer, 2006.
4. J. R. Munkres: Topology, Pearson, 2nd edition, 2000.

MAT 3000: Elementary Differential Geometry

Syllabus: Space curves, Curvature and orientability of surfaces, Gauss-Bonnet theorem, Brief introduction to metric geometry Hopf-Rinow theorem.

Prerequisites: Calculus, Multivariable Calculus, Real Analysis, Metric and topological spaces.

References:

1. M P. do Carmo: Differential Geometry of Curves and Surfaces, Prentice-Hall, (1976).

2. A. Pressley: Elementary Differential Geometry, Springer, 2010.

MAT 3013: Mathematical Modeling (Differential Equations)

Syllabus: Differential equation associated to real life problems, First order differential equation on R of the form $y'(x) = f(x, y(x))$, Equivalent integral equation, Existence of approximate solutions of equation upto error ϵ by the Cauchy-Euler method, Existence and uniqueness of solutions when f is Lipshitz continuous in the second variable, Necessary conditions for $f(x, y)$ to be Lipshitz continuous in y, Picard's method of solutions of equation, Higher order differential equations, Vector valued ordinary differential equations, Reformulation of higher order differential equations as first order vector valued differential equations, Linear vector valued first order differential equation, $Y'(x) = A\,Y(x) + C(x)$ — Homogeneous case, $C = 0$, Characteristic values, characteristic vectors of square matrices, Solution when A is independent of x, Linear independence of solutions associated to characteristic values, General solution of the inhomogeneous equation, Peano's approximation method for existence of solution.

Prerequisites: Calculus, Multivariable calculus, Real analysis, Linear algebra.

References:

1. E. A. Coddington: An Introduction to ordinary differential equations, Prentice Hall India, 1968
2. V. I. Arnold: Ordinary Differential Equations, MIT Press.

MAT 3018: Complex Analysis

Syllabus: The algebra and geometry of complex numbers, representations of a complex number. Exponential and logarithmic functions. Differentiation, analytic functions, Cauchy-Riemann equations. Contour integrals; Independence of path. Cauchy's Integral Theorem, Cauchy's Integral Formula, Liouville's Theorem and its applications. Complex power series, uniform convergence. Removable and isolated singularities. Taylor's and Laurent's Theorems. The residue theorem and applications.

Prerequisites: Calculus, Multivariable calculus, Real analysis, Linear algebra.

References:

1. L. V. Ahlfors: Complex Analysis, McGraw Hill, 1979.
2. J. B. Conway: Functions of one complex variable, Springer international students' edition.
3. T. W. Gamelin: Complex Analysis, Springer, 2003.

MAT 3120: Linear Algebra and Matrix Analysis

Syllabus: Bases, dimension. Subspaces. Norms and inner products. Linear operators. Matrix representations. Similarity and unitary similarity. Dual spaces. Transpose and adjoint. Eigenvalues, singular values and norms of operators. Special classes of operators: hermitian, normal, unitary, positive definite, projections. Spectral theorem. Singular value decomposition. Schur triangular form. QR decomposition. Applications. Commuting operators and simultaneous reduction to diagonal and triangular forms. Additional topics to be chosen from the following (suggested) list: Variational principles for eigenvalues and singular values, The Jordan canonical form; nonnegative matrices and the Perron Frobenius theory; applications of singular value decomposition, discrete Fourier transform.

Prerequisites: Calculus, Real Analysis, Linear algebra.
References:

1. S. Axler: Linear Algebra Done Right, Second Edition, UTM, Springer, 1997
2. M. E. Taylor, Linear Algebra.
3. S. R. Garcia and R. Horn: A Second Course in Linear Algebra.